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On Radio k - Chromatic Number of Splitting and Shadow graph of Various Graphs

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Abstract

Radio k- Coloring of graph T is a simply connected graph with $diam(T)$ and a positive integer k, $1 \leq k \leq diam(T)$, a radio k- coloring of T is an allocation ϕ of positive integers to the vertices of T such that $|\phi(a) - \phi(b)| \geq 1 + k - d(a, b)$, where a and b are any two different vertices of T and $d(a, b)$ is the interval between a and b. The supreme color granted by ϕ is called span, denoted by $rc_k(\phi)$. The radio k - chromatic number $rc_k(T)$ of T is the minimal $rc_k(\phi)$, where ϕ is a radio k- coloring of T. In this paper, we obtain the exact value of the radio k- chromatic number for splitting graph and shadow graph of Star graph $K_{1,n}$ and Double Star graph $K_{1,n,n}$.

Keywords: *Radio k - Coloring, Radio k- Chromatic Number, Star graph and Double Star graph.*

2010 Mathematics Subject Classification: 05C15.

1 Introduction

Graph theory is a branch of Mathematics and Computer science that studies graphs, which are structures consisting of vertices (or nodes) and edges that connect pairs of vertices. Graphs can be directed or undirected, and they are used to model pairwise relations between objects. Graph theory has many real-world applications, including computer networks, social networks, transportation systems, and more. It is a powerful tool in algorithm design, network analysis, optimization, and other fields.

Graph coloring is one of the fundamental concepts in graph theory, which involves assigning colors to vertices of a graph such that no adjacent vertices have the same color. The problem of graph coloring has wide-ranging applications in computer science, operations research, Physics, Chemistry, Biology, and many other fields. In 1941, Brooks coloring the nodes of a network and which developed to many of the following coloring.

Vertex coloring is the most common type of coloring in which each vertex is assigned a color such that no two adjacent vertices have the same color. The minimum number of colors required to color a graph is called its chromatic number [1].

Graph coloring problem is a NP Complete problem. Griggs and Yeh[2] adapted this to graphs as follows: For non-negative integers $p_1, p_2, ..., p_m$, an $L(p_1, p_2, ..., p_m)$ - coloring of a graph T is a coloring of its vertices by non - negative integers such that vertices at interval exactly i get colors that vary by at least p_i. The supreme color grant to any node is called the span of the coloring. The main objective of the problem is to raise an $L(p_1, p_2, ..., p_m)$ -coloring of the minimal span. Chartrand et al.[3] introduced a variation $L(p_1, p_2, ..., p_m)$ - coloring known as radio k - coloring of graphs. Chartrand et al.[3] have studied radio number of some known graphs, like cycles, complete multipartite graphs, and graphs with diameter 2 and characterization of connected graphs of order n and diameter 2. When $k = diam(T)$, the radio k-chromatic number is called the radio number and it is denoted by $rn(G)$. When $k = 2$, the radio k-chromatic number is called radio 2-coloring is identical to $L(2, 1)$ labeling and When $k = 1$, the radio k-chromatic number is called radio 1-coloring is equivalent to vertex coloring of graphs. Orden, and Gimenez [4], gives the Spectrum Graph Coloring and Applications to Wi-Fi Channel Assignment. Nirajan and Srinivasa Rao Kola [5] discussed, the radio k-chromatic number of paths. The upper bound for radio k chromatic number were determined by the authors Laxman Saha, [6] and Elsayed, Badr [7]. The lower bound for radio k chromatic number were discussed by the authors Sandip Das, Sasthi [8] and Nirajan and Srinivasa Rao Kola [9].

In this paper, we determine the radio k- chromatic number for splitting graph and shadow graph of Star graph $K_{1,n}$ and Double Star graph $K_{1,n,n}$.

2 Preliminaries

For any graph T we denote its set of vertices and set of edges by $V(T)$ and $E(T)$, respectively. The distance $d(a, b)$ between two vertices a and b of a graph T is the length (number of edges) of a shortest path connecting a and b. The diameter $diam(T)$ of a graph T is the maximum $d(a,b)$ taken over every pair of vertices a,b of T. For any positive integer $1 \leq k \leq diam(T)$, *radio k-coloring* [2] of T is a function $\phi : V(T) \to \{i; 1 \leq i \leq n\}$ satisfying the condition $|\phi(a) - \phi(b)| \geq 1 + k - d(a,b)$, for all distinct $a, b \,\epsilon\, V(T)$. The radio k-chromatic number $rc_k(T)$ of T is the minimum $rc_k(\phi)$. *star graph* $K_{1,n}$ is the complete bipartite graph, a tree with one internal node and k leaves. A tree containing exactly two non-pendant vertices is called a *double star graph* and it is denoted by $K_{1,m,m}$. For each vertex v of a graph T, take a new vertex v. Join the vertex v to all the vertices of T adjacent to the vertex v. The graph $S(T)$ thus obtained is called *splitting graph*[10] of T. The *shadow graph* $D(T)$ of a connected graph T is constructed by taking two copies of T say T' and T". Join each vertex u in T' to the neighbours of the corresponding vertex u in T".

Lemma 2.1 *For any graph T , we have*

$$rc_k(T) \geq \begin{cases} |D_K| - 2p + 2\sum_{i=0}^{1}{}_{i=0}^{p}|L_i|(p-i), & when \quad k = 2p+1 \\ |D_K| - 2p + 2\sum_{i=0}^{1}{}_{i=0}^{p}|L_i|(p-i) + 1, & when \quad k = 2p \end{cases}$$

Here let T be a graph. We describe a general technique and prove a formula to compute a lower bound for $rc_k(T)$ where $1 \leq k \leq diam(T)$. We start by defining the following sets depending on the parity of k. For $k = 2p$ an even integer, pick a vertex v of T and let $L_0 = v$. For $k = 2p+1$ an odd integer, pick a maximal clique C of T and let $L_0 = C$. For any $S \subseteq V(T)$, let $N(S)$ denote the set of vertices adjacent to at least one vertex of S. Now recursively define $L_{i+1} = N(L_i) \backslash (L_0 \bigcup L_1 \bigcup ... \bigcup L_i)$ for all $i \in 0, 1, ..., p-1$. These sets are called layers and in particular L_i is called the ith layer. Note that for any T with $diam(T) \geq k$ and any choice of L_0 the sets $L_i \neq$ for all $i = 0, 1, ..., p$. Let D_k be the subgraph of T induced by the vertices of $L_0 \bigcup L_1 \bigcup ... \bigcup L_p$. Note that $diam(D_k) \leq k$.

3 Main results

In this section, we obtain the exact value of the radio k- chromatic number for splitting graph of Star graph $K_{1,n}$ and Double Star graph $K_{1,n,n}$ and shadow graph of Star graph $K_{1,n}$ and Double Star graph $K_{1,n,n}$.

Theorem 3.1
For any positive integer $n \geq 2$ and $2 \leq k \leq diam(S(K_{1,n}))$, then the radio k-chromatic number of splitting graph of star graph is

$$rc_k(S(K_{1,n})) = \begin{cases} 2n+1, & k = diam(S(K_{1,n})) - 1 \\ 4n+1, & k = diam(S(K_{1,n})) \end{cases}$$

Proof:

Let the vertex set and edge set of star graph is defined by $V(K_{1,n}) = \{v_i : 1 \leq i \leq n+1\}$ and $E(K_{1,n}) = \{v_1 v_i : 2 \leq i \leq n+1\}$ with $|V(K_{1,n})| = n+1, |E(K_{1,n})| = n$ and $diam(K_{1,n}) = 2$.

By the definition of splitting graph, the vertex set of splitting graph of star graph is defined by $V(S(K_{1,n})) = \{v_i : 1 \leq i \leq n+1\} \cup \{u_i : 1 \leq i \leq n+1\}$ with $|V(K_{1,n})| = 2n+2$ and $diam(S(K_{1,n})) = 3$.

Case 1: when $k = diam(S(K_{1,n})) - 1$

From Lemma 2.1, we have $rc_k(S(K_{1,n})) \geq |D_2| - 2 + 2\sum_{i=0}^{1}(|L_i|(1-i)) + 1$, so that $rc_k(S(K_{1,n})) \geq 2n+1$

In case, if the radio k chromatic number consists less than 2n+1, it does not satisfy the radio k coloring condition $|\phi(a) - \phi(b)| \geq 1 + k - d(a,b)$.

Consider the coloring mapping $\phi : V(S(K_{1,n})) \rightarrow \{0, 1, 2, ..., 2n+1\}$.

Assign the colors for the vertices of $V(S(K_{1,n}))$ as follows.

$\phi(v_1) = 0$

$\phi(u_i) = i$, where $1 \leq i \leq n+1$

$\phi(v_{i+1}) = n+1+i$, where $1 \leq i \leq n$

Hence the above assignment of colors satisfies the condition of radio k-coloring. we have that $rc_k(S(K_{1,n})) \leq 2n+1$

Therefore, we get $rc_k(S(K_{1,n})) = 2n+1$

Case 2: when $k = diam(S(K_{1,n}))$

From Lemma 2.1, we have $rc_k(S(K_{1,n})) \geq |D_3| - 2 + 2\sum_{i=0}^{1}(|L_i|(1-i))$, so that $rc_k(S(K_{1,n})) \geq 4n+1$

In case, if the radio k chromatic number consists less than 4n+1, it does not satisfy the radio k coloring condition $|\phi(a) - \phi(b)| \geq 1 + k - d(a,b)$.

Consider the coloring mapping $\phi : V(S(K_{1,n})) \rightarrow \{0, 1, 2, ..., 4n+1\}$.

Assign the colors for the vertices of $V(S(K_{1,n}))$ as follows.

$\phi(v_1) = 0$

$\phi(u_1) = 2$

$\phi(u_{i+1}) = 2i+1$, where $1 \leq i \leq n$

$\phi(v_{i+1}) = 2n+1+2i$, where $1 \leq i \leq n$

Hence the above assignment of colors satisfies the condition of radio k-coloring. We have that $rc_k(S(K_{1,n})) \leq 4n+1$

Therefore, we get $rc_k(S(K_{1,n})) = 4n+1$

Theorem 3.2

For any positive integer $n \geq 2$ and $2 \leq k \leq diam(D(K_{1,n}))$, then the radio k-chromatic number of shadow graph of star graph is $rc_k(D(K_{1,n})) = (2k-2)n+k$

Proof:

Let the vertex set and edge set of star graph is defined by $V(K_{1,n}) = \{v_i : 1 \leq i \leq n+1\}$ and $E(K_{1,n}) = \{v_1 v_i : 2 \leq i \leq n+1\}$ with $|V(K_{1,n})| = n+1, |E(K_{1,n})| = n$ and $diam(K_{1,n}) = 2$.

By the definition of shadow graph, the vertex set of shadow graph of star graph is defined by $V(D(K_{1,n})) = \{v_i : 1 \leq i \leq n+1\} \cup \{u_i : 1 \leq i \leq n+1\}$ with $|V(K_{1,n})| = 2n+2$ and $diam(D(K_{1,n})) = 3$.

Case 1: when $k = diam(D(K_{1,n})) - 1$

From Lemma 2.1, we have $rc_k(K_{1,n}) \geq |D_2| - 2 + 2\sum_{i=0}^{1}(|L_i|(1-i)) + 1$, so that $rc_k(D(K_{1,n})) \geq (2k-2)n + k$

In case, if the radio k chromatic number consists less than $(2k-2)n+k)$, it does not satisfy the radio k coloring condition $|\phi(a) - \phi(b)| \geq 1 + k - d(a,b)$. Consider the coloring mapping $\phi : V(D(K_{1,n})) \to \{0, 1, 2, ..., (2k-2)n+k\}$. Assign the colors for the vertices of $V(D(K_{1,n}))$ as follows.

$\phi(v_1) = 0$
$\phi(u_1) = 2$
$\phi(v_{i+1}) = 2 + i$, where $1 \leq i \leq n$
$\phi(u_{i+1}) = n + 2 + i$, where $1 \leq i \leq n$

Hence the above assignment of colors satisfies the condition of radio k-coloring. we have that $rc_k(D(K_{1,n})) \leq (2k-2)n + k$

Therefore, we get $rc_k(D(K_{1,n})) = (2k-2)n + k$

Case 2: when $k = diam(D(K_{1,n}))$

From Lemma 2.1, we have $rc_k(D(K_{1,n})) \geq |D_3| - 2 + 2\sum_{i=0}^{1}(|L_i|(1-i)) + 1$, so that $rc_k(D(K_{1,n})) \geq (2k-2)n + k$

In case, if the radio k chromatic number consists less than $(2k-2)n+k$, it does not satisfy the radio k coloring condition $|\phi(a) - \phi(b)| \geq 1 + k - d(a,b)$. Consider the coloring mapping $\phi : V(D(K_{1,n})) \to \{0, 1, 2, ..., (2k-2)n+k\}$. Assign the colors for the vertices of $V(D(K_{1,n}))$ as follows.

$\phi(v_1) = 0$
$\phi(u_1) = 2$
$\phi(v_{i+1}) = 3 + 2i$, where $1 \leq i \leq n$
$\phi(v_{i+1}) = 2n + 3 + 2i$, where $1 \leq i \leq n$

Hence the above assignment of colors satisfies the condition of radio k-coloring. we have that $rc_k(D(K_{1,n})) \leq (2k-2)n + k$

Therefore, we get $rc_k(D(K_{1,n})) = (2k-2)n + k$

Theorem 3.3

For any positive integer $n \geq 2$ and $2 \leq k \leq diam(S(K_{1,n,n}))$ then the radio k-chromatic number of splitting graph of double star graph is,

$$rc_k(S(K_{1,n,n})) = \begin{cases} (k-1)n + (2k+1), & k = diam(S(K_{1,n,n})) - 2 \ \& \ diam(S(K_{1,n,n})) - 1 \\ 6n + 5, & k = diam(S(K_{1,n,n})) \end{cases}$$

Proof:

Let the vertex set and edge set of double star graph is defined by $V(K_{1,n.n}) = \{v_i : 1 \leq i \leq 2n+1\}$ and $(E(K_{1,n.n})) = \{v_1 v_{i+1} : 1 \leq i \leq n\} \cup \{v_{i+1} v_{n+i+1} : 1 \leq i \leq n\}$ with $|V(K_{1,n,n})| = 2n+1$, $|E(K_{1,n,n})| = 2n$ and $diam(K_{1,n,n}) = 3$.

By the definition of splitting graph, the vertex set of splitting graph of double star graph is defined by $V(S(K_{1,n,n})) = \{v_i : 1 \leq i \leq 2n+1\} \cup \{u_i : 1 \leq i \leq 2n+1\}$ with $|V(S(K_{1,n,n}))| = 2n+2$ and $diam(S(K_{1,n,n})) = 4$.

Case 1:

Subcase 1: when $k = diam(S(K_{1,n,n})) - 2$

From Lemma 2.1, we have $rc_k(S(K_{1,n,n})) \geq |D_2| - 2 + 2\sum_{i=0}^{1}(|L_i|(1-i)) + 1$, so that $rc_k(S(K_{1,n,n})) \geq (k-1)n + (2k+1)$

In case, if the radio k chromatic number consists less than $(k-1)n + (2k+1)$, it does not satisfy the radio k coloring condition $|\phi(a) - \phi(b)| \geq 1 + k - d(a,b)$.

Consider the coloring mapping $\phi : V(S(K_{1,n,n})) \rightarrow \{0, 1, 2, ..., (k-1)n + (2k+1)\}$.

Assign the colors for the vertices of $V(S(K_{1,n,n}))$ as follows.

$\phi(v_1) = 0$

$\phi(u_1) = 2$

$\phi(u_{n+1+i}) = 4,$ where $1 \leq i \leq n$

$\phi(u_{i+1}) = 1 + 2i,$ where $1 \leq i \leq n$

$\phi(v_{n+i+1}) = 4 + 2i,$ where $1 \leq i \leq n$

$\phi(v_{i+1}) = 7 + 2i,$ where $1 \leq i \leq n$

Hence the above assignment of colors satisfies the condition of radio k-coloring. we have that $rc_k(S(K_{1,n,n})) \leq (k-1)n + (2k+1)$

Therefore, we get $rc_k(S(K_{1,n,n})) = (k-1)n + (2k+1)$

Subcase 2: when $k = diam(S(K_{1,n,n})) - 1$

From Lemma 2.1, we have $rc_k(S(K_{1,n,n})) \geq |D_3| - 2 + 2\sum_{i=0}^{1}(|L_i|(1-i))$, so that $rc_k(S(K_{1,n,n})) \geq (k-1)n + (2k+1)$

In case, if the radio k chromatic number consists less than $(k-1)n + (2k+1)$, it does not satisfy the radio k coloring condition $|\phi(a) - \phi(b)| \geq 1 + k - d(a,b)$.

Consider the coloring mapping $\phi : V(S(K_{1,n,n})) \rightarrow \{0, 1, 2, ..., (k-1)n + (2k+1)\}$.

Assign the colors for the vertices of $V(S(K_{1,n,n}))$ as follows.

$\phi(v_1) = 0$

$\phi(u_1) = 2$

$\phi(u_{n+1+i}) = 4,$ where $1 \leq i \leq n$

$\phi(v_{n+i+1}) = 4 + 2i,$ where $1 \leq i \leq n$

$\phi(v_{i+1}) = 7 + 2i,$ where $1 \leq i \leq n$

$\phi(u_{i+1}) = 1 + 2i,$ where $1 \leq i \leq n$

Hence the above assignment of colors satisfies the condition of radio k-coloring. we have that $rc_k(S(K_{1,n,n})) \leq (k-1)n + (2k+1)$

Therefore, we get $rc_k(S(K_{1,n,n})) = (k-1)n + (2k+1)$

Case 2: when $k = diam(S(K_{1,n,n})$

From Lemma 2.1, we have $rc_k(S(K_{1,n,n})) \geq |D_4| - 4 + 2\sum_{i=0}^{2}(|L_i|(1-i)) + 1$, so that $rc_k(S(K_{1,n,n})) \geq 6n + 5$

In case, if the radio k chromatic number consists less than 6n+5, it does not satisfy the radio k coloring condition $|\phi(a) - \phi(b)| \geq 1 + k - d(a, b)$.

Consider the coloring mapping $\phi : V(S(K_{1,n,n})) \to \{0, 1, 2, ..., 6n + 5\}$.

Assign the colors for the vertices of $V(S(K_{1,n,n}))$ as follows.

$\phi(v_1) = 0$

$\phi(u_1) = 3$

$\phi(u_{n+1+i}) = 6,$ where $1 \leq i \leq n$

$\phi(v_{n+i+1}) = 6 + 3i,$ where $1 \leq i \leq n$

$\phi(v_{i+1}) = 3n + 3i + 5,$ where $1 \leq i \leq n$

$\phi(u_{i+1}) = 2 + 3i,$ where $1 \leq i \leq n$

Hence the above assignment of colors satisfies the condition of radio k-coloring. we have that $rc_k(S(K_{1,n,n})) \leq 6n + 5$

Therefore, we get $rc_k(S(K_{1,n,n})) = 6n + 5$

Theorem 3.4

For any positive integer $n \geq 2$ and $2 \leq k \leq diam(D(K_{1,n,n}))$, then the radio k-chromatic number of shadow graph of double star graph is,

$$rc_k(D(K_{1,n,n})) = \begin{cases} 2n+3, & k = diam(D(K_{1,n,n})) - 2 \\ (2k-2)n + (3k-2), & k = diam(D(K_{1,n,n})) - 1 \ \& \ diam(D(K_{1,n,n})) \end{cases}$$

Proof:

Let the vertex set and edge set of double star graph is defined by $V(K_{1,n.n}) = \{v_i : 1 \leq i \leq 2n+1\}$ and $E(K_{1,n.n}) = \{v_1 v_{i+1} : 1 \leq i \leq n\} \cup \{v_{i+1} v_{n+i+1} : 1 \leq i \leq n\}$ with $|V(K_{1,n,n})| = 2n + 1$, $|E(K_{1,n,n})| = 2n$ and $diam(K_{1,n,n}) = 3$.

By the definition of Shadow graph, the vertex set of shadow graph of double star graph is defined by $V(D(K_{1,n,n})) = \{v_i : 1 \leq i \leq 2n + 1\} \cup \{u_i : 1 \leq i \leq 2n + 1\}$ with $|VD((K_{1,n,n}))| = 2n + 2$ and $diam(D(K_{1,n,n})) = 4$.

Case 1: when $k = diam(D(K_{1,n,n})) - 2$

From Lemma 2.1, we have $rc_k(D(K_{1,n,n})) \geq |D_2| - 2 + 2\sum_{i=0}^{1}(|L_i|(1-i)) + 1$, so that $rc_k(D(K_{1,n,n})) \geq 2n + 3$

In case, if the radio k chromatic number consists less than 2n+3, it does not satisfy the radio k coloring condition $|\phi(a) - \phi(b)| \geq 1 + k - d(a, b)$.

Consider the coloring mapping $\phi : V(D(K_{1,n,n})) \to \{0, 1, 2, ..., 2n + 3\}$.

Assign the colors for the vertices of $V(D(K_{1,n,n}))$ as follows.

$\phi(v_1) = 0$

$\phi(u_1) = 1$

$\phi(v_{n+1+i}) = 2,$ where $1 \leq i \leq n$

$\phi(u_{n+1+i}) = 3,$ where $1 \leq i \leq n$

$\phi(v_{i+1}) = 3 + i,$ where $1 \leq i \leq n$

$\phi(u_{i+1}) = n + 3 + i$, where $1 \leq i \leq n$

Hence the above assignment of colors satisfies the condition of radio k-coloring.
we have that $rc_k(D(K_{1,n,n})) \leq 2n + 3$

Therefore, we get $rc_k(D(K_{1,n,n})) = 2n + 3$

Case 2:

Subcase 1: when $k = diam(D(K_{1,n,n})) - 1$

From Lemma 2.1, we have $rc_k(D(K_{1,n,n})) \geq |D_3| - 2 + 2\sum_{i=0}^{1}(|L_i|(1-i))$,
so that $rc_k(D(K_{1,n,n})) \geq (2k-2)n + (3k-2)$

In case, if the radio k chromatic number consists less than $(2k-2)n+(3k-2)$, it does not satisfy the radio k coloring condition $|\phi(a)-\phi(b)| \geq 1+k-d(a,b)$.
Consider the coloring mapping $\phi : V(D(K_{1,n,n})) \to \{0, 1, 2, ..., (2k-2)n + (3k-2)\}$.
Assign the colors for the vertices of $V(D(K_{1,n,n}))$ as follows.

$\phi(v_1) = 0$

$\phi(u_1) = 2$

$\phi(v_{n+1+i}) = 4$, where $1 \leq i \leq n$

$\phi(u_{n+1+i}) = 6$, where $1 \leq i \leq n$

$\phi(v_{i+1}) = 7 + 2i$, where $1 \leq i \leq n$

$\phi(u_{i+1}) = 2n + 7 + 2i$, where $1 \leq i \leq n$

Hence the above assignment of colors satisfies the condition of radio k-coloring.
we have that $rc_k(D(K_{1,n,n})) \leq (2k-2)n + (3k-2)$

Therefore, we get $rc_k(D(K_{1,n,n})) = (2k-2)n + (3k-2)$

Subcase 2: when $k = diam(D(K_{1,n,n}))$

From Lemma 2.1, we have $rc_k(D(K_{1,n,n})) \geq |D_4| - 4 + 2\sum_{i=0}^{2}(|L_i|(1-i)) + 1$,
so that $rc_k(D(K_{1,n,n})) \geq (2k-2)n + (3k-2)$

In case, if the radio k chromatic number consists less than $(2k-2)n+(3k-2)$, it does not satisfy the radio k coloring condition $|\phi(a)-\phi(b)| \geq 1+k-d(a,b)$.
Consider the coloring mapping $\phi : V(D(K_{1,n,n})) \to \{0, 1, 2, ..., (2k-2)n + (3k-2)\}$.
Assign the colors for the vertices of $V(D(K_{1,n,n}))$ as follows.

$\phi(v_1) = 0$

$\phi(u_1) = 3$

$\phi(v_{n+1+i}) = 6$, where $1 \leq i \leq n$

$\phi(u_{n+1+i}) = 9$, where $1 \leq i \leq n$

$\phi(v_{i+1}) = 10 + 3i$, where $1 \leq i \leq n$

$\phi(u_{i+1}) = 3n + 10 + 3i$, where $1 \leq i \leq n$

Hence the above assignment of colors satisfies the condition of radio k-coloring.
we have that $rc_k(D(K_{1,n,n})) \leq (2k-2)n + (3k-2)$

Therefore, we get $rc_k(D(K_{1,n,n})) = (2k-2)n + (3k-2)$

4 Conclusion

In this paper, we attain the exact value of the radio k- chromatic number for splitting graph and shadow graph of Star graph $K_{1,n}$ and Double Star graph $K_{1,n,n}$. Despite of the few results in radio coloring we still experiment on radio k- chromatic number of various families of graphs.

5 Open Problem

However, the above results can be the foundation step to develop the general bound to find the radio k- chromatic number for splitting graph and shadow graph of any graph T, which is still an open problem.

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