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Dominator and Total Dominator Chromatic Number of Duplication Corresponding Corona of Some Graphs

Megala. S[1], Mohanapriya. N[2] & Karthika. R[3]

[1,2,3] PG and Research Department of Mathematics,
Kongunadu Arts and Science College,
Coimbatore - 641029, Tamilnadu, India
e-mail: smegalarave@gmail.com, phdmohana@gmail.com, karthika.20.r@gmail.com

Abstract

In this paper, we have considered two graphs $\mathscr{G}_1$ and $\mathscr{G}_2$ to be any two vertex disjoint graphs with n_1 and n_2 vertices respectively. Let $V(\mathscr{G}_1) \cup$ U $(\mathscr{G}_1)$ where, $V(\mathscr{G}_1) = \{v_1, v_2, v_3, ..., v_{n_1}\}$ and $U(\mathscr{G}_1) = \{x_1, x_2, x_3, , ..., x_{n_1}\}$ be the vertex set of $\mathscr{D}\mathscr{G}_1$, the duplication graph of $\mathscr{G}_1$. The vertex x_i is a duplication of v_i for each i = 1,2,..., n_1. Duplication corresponding corona of $\mathscr{G}_1$ and $\mathscr{G}_2$, denoted by $\mathscr{G}_1 \circledast \mathscr{G}_2$ is the graph obtained from $\mathscr{D}\mathscr{G}_1$ and n_1 copies of $\mathscr{G}_2$ by making x_i and v_i adjacent to every vertex in the i - th copy of $\mathscr{G}_2$. Our aim is to obtain the results on dominator and total dominator coloring of duplication corresponding corona of some graphs.

Keywords: *Dominator chromatic number, Total dominator chromatic number, Duplication corresponding corona.*

Mathematics Subject Classification: $05C15$

1 Introduction

Graph theory is becoming interestingly significant as it is being actively applied in biochemistry, nanotechnology, electrical engineering, computer science, and operations research. The powerful combinatorial method found in graph theory has also been used to prove the results of pure mathematics. The origin of graph theory started with the problem of Konigsberg bridge, in 1735.Euler studied the problem of Konigsberg bridge and constructed a structure to solve the problem called Eulerian graph[7]. In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems[7].

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as vertices, and the links that connect the vertices are called edges. A graph is a pair (V,E), where V is a finite set and E is a binary relation on V. V is called a vertex set whose elements are called vertices. E is a collection of edges, where an edge is a pair (u,v) with u,v in V. Graphs are one of the prime objects of study in discrete mathematics. Certain discrete problems can be profitably analyzed using graph theoretic methods. A Proper coloring[1, 2, 4, 10] is a transformation $f : V(\mathscr{G}) \rightarrow C$ in which any two adjacent vertices have distinct colors, where f is the function, $V(\mathscr{G})$ denotes the set of vertices and C denotes the collection of color classes.

A dominator coloring of a graph $\mathscr{G}$ is also one of the proper coloring such that each vertex of the graph $\mathscr{G}$, is in the closed neighbourhood of every vertex of atleast a color class. The minimum number of color classes needed for dominator of $\mathscr{G}$ is known as Dominator chromatic number [5, 8, 9, 11, 15]. A Total dominator coloring [3] of a graph $\mathscr{G}$ is a proper coloring such that every vertex dominates atleast one color class other than its own. The minimum number of color classes needed for a total dominator coloring of $\mathscr{G}$ is known as the Total dominator chromatic number. Let $\mathscr{G}_1$ and $\mathscr{G}_2$ be the two vertex disjoint graphs with n_1 and n_2 vertices respectively. Let $V(\mathscr{G}_1) \cup$ U $(\mathscr{G}_1)$ where $V(\mathscr{G}_1) = \{v_1, v_2, v_3, ..., v_{n_1}\}$ and $U(\mathscr{G}_1) = \{x_1, x_2, x_3, , ..., x_{n_1}\}$ be the vertex sets of $\mathscr{D}\mathscr{G}_1$, the duplication graph of $\mathscr{G}_1$. The vertex x_i is a duplication of v_i for each i = 1,2,..., n_1. Duplication corresponding corona [6, 12, 13, 14] of $\mathscr{G}_1$ and $\mathscr{G}_2$, denoted by $\mathscr{G}_1 \divideontimes \mathscr{G}_2$ is the graph obtained from $\mathscr{D}\mathscr{G}_1$ and n_1 copies of $\mathscr{G}_2$ by making x_i and v_i adjacent to every vertex in the i - th copy of $\mathscr{G}_2$ for $i = 1, 2, ..., n_1$.

In this paper, we compute the dominator and total dominator chromatic number of the duplication corresponding corona of path, cycle and wheel graphs.

2 Preliminaries

This section deals with some of the preliminiary results and bounds that are required for our study.

Lemma 2.1 *[5]*
Let $\mathscr{G}$ be a connected graph. Then

$$max\ \{\chi(G), \gamma(G)\} \leq \chi_d(G) \leq \chi(G) + \gamma.$$

Lemma 2.2 *[3]*
For any connected graph G of order n with $\delta(G) \geq 1$. Then

$$max\ \{\chi(G), \gamma_t(G), 2\} \leq \chi_d^t(G) \leq n.$$

Proposition 2.1 *[1, 5] For the path P_n , $n \geq 2$, we have*

$$\chi_d(P_n) = \begin{cases} \lceil \frac{n}{3} \rceil + 1, & when \ \ n = 2, 3, 4, 5, 7 \\ \lceil \frac{n}{3} \rceil + 2, & otherwise. \end{cases}$$

Proposition 2.2 *[3] Let P_n be a path of order $n \geq 2$. Then*

$$\chi_d^t(P_n) = \begin{cases} 2 \lceil \frac{n}{3} \rceil - 1, & if\ n \equiv 1 (mod\ 3) \\ 2 \lceil \frac{n}{3} \rceil, & otherwise. \end{cases}$$

Proposition 2.3 *[1, 5] For the cycle C_n , we have*

$$\chi_d(C_n) = \begin{cases} \lceil \frac{n}{3} \rceil, & when \ \ n = 4 \\ \lceil \frac{n}{3} \rceil + 1, & when\ n = 5 \\ \lceil \frac{n}{3} \rceil + 2, & otherwise. \end{cases}$$

Proposition 2.4 *[3] Let C_n be a cycle of order $n \geq 3$. Then*

$$\chi_d^t(C_n) = \begin{cases} 2 & if\ n = 4, \\ 4\lfloor \frac{n}{6} \rfloor + r, & if\ n \neq 4\ and\ for\ r = 0, 1, 2, 4,\ n \equiv r (mod\ 6) \\ 4\lfloor \frac{n}{6} \rfloor + r - 1, & if\ n \equiv r (mod\ 6),\ where\ r = 3, 5 \end{cases}$$

Proposition 2.5 *[11] The wheel $W_{1,n}$ has*

$$\chi_d(W_{1,n}) = \begin{cases} 3, & if\ n\ is\ even \\ 4, & if\ n\ is\ odd. \end{cases}$$

Proposition 2.6 *[3] Let W_n be a wheel of order $n + 1 \geq 4$. Then*

$$\chi_d^t(W_n) = \begin{cases} 3 & if\ n\ is\ even \\ 4 & if\ n\ is\ odd \end{cases}$$

3 Main results

In this section, we determine the dominator and total dominator chromatic number for duplication corresponding corona of Path with path, Path with cycle, Cycle with path, Cycle with cycle denoted as $\chi_d(P_n \circledast P_m)$, $\chi_d^t(P_n \circledast P_m)$, $\chi_d(P_n \circledast C_m)$, $\chi_d^t(P_n \circledast C_m)$, $\chi_d(C_n \circledast P_m)$, $\chi_d^t(C_n \circledast P_m)$, $\chi_d(C_n \circledast C_m)$, $\chi_d^t(C_n \circledast C_m)$ and Path with wheel, Wheel with path, Wheel with wheel graphs as $\chi_d(P_n \circledast W_{1,m})$, $\chi_d^t(P_n \circledast W_{1,m})$, $\chi_d(W_{1,n} \circledast P_m)$, $\chi_d^t(W_{1,n} \circledast P_m)$, $\chi_d(W_{1,n} \circledast W_{1,m})$, $\chi_d^t(W_{1,n} \circledast W_{1,m})$.

Theorem 3.1 *For any integers $n, m \geq 2$, the dominator chromatic number of duplication corresponding corona of path P_n with path P_m is*

$$\chi_d(P_n \circledast P_m) = n + 3$$

proof Let P_n be the path graph with vertex set $V(P_n)$ and $\mathscr{D}P_n$ denote the duplication of the graph P_n. The vertices in the duplication graph are defined as

$$V(\mathscr{D}P_n) = \{v_a, x_a : 1 \leq a \leq n\}$$

Let us consider 'n' copies of another path P_m, whose vertices are defined as,

$$V(P_m) = \{x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m\}$$

Let 'i' denote the respective copy of P_m, where $1 \leq i \leq n$, so that the vertex set of the duplication corresponding corona graph is given by,

$$V(P_n \circledast P_m) = \{v_a, x_a, x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m\}$$

The cardinality of the vertex set of the resultant graph is given by $\mid V(P_n \circledast P_m) \mid= n(m + 2)$. The maximum and minimum degree of the graph are $\Delta\ (P_n \circledast P_m) = m + 2$ and $\delta\ (P_n \circledast P_m) = 3$ respectively.

Let the coloring be represented as $C : V(P_n \circledast P_m) \rightarrow$ {1,2,...,n + 3}
We obtain the dominator coloring as follows: For $1 \leq a \leq n$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1 (mod\ 2) \\ 3, & for\ b \equiv 0 (mod\ 2) \end{cases}$$

$$C(x_a) = a + 3$$

- Allocation of distinct colors to the vertices x_a makes it possible for other vertices v_a and x_{ab} in every i - th copy to dominate the vertices x_a. These x_a are inturn self - dominating.

- Usage of more number of colors than n+3, say if, $\chi_d(P_n \circledast P_m) \geq n+3$, will provide a maximum dominator chromatic number. Using colors less than n+3, say if, $\chi_d(P_n \circledast P_m) \leq n+3$, will not satisfy the dominator coloring property.

Hence we require n + 3 colors.

$$\chi_d(P_n \circledast P_m) = n+3$$

Theorem 3.2 *For any integers $n, m \geq 2$, the total dominator chromatic number of duplication corresponding corona of path P_n with path P_m is*

$$\chi_d^t(P_n \circledast P_m) = 2n+2$$

proof Let P_n be the path graph with vertex set $V(P_n)$ and $\mathscr{D}P_n$ denote the duplication of the graph P_n. The vertices in the duplication graph are defined as

$$V(\mathscr{D}P_n) = \{v_a, x_a : 1 \leq a \leq n\}$$

Let us consider 'n' copies of another path P_m, whose vertices are defined as,

$$V(P_m) = \{x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m\}$$

Let 'i' denote the respective copy of P_m, where $1 \leq i \leq n$, so that the vertex set of the duplication corresponding corona graph is given by,

$$V(P_n \circledast P_m) = \{v_a, x_a, x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m\}$$

The cardinality of the vertex set and the degrees are followed from the previous theorem.

Let the coloring be represented as $C : V(P_n \circledast P_m) \rightarrow$ {1,2,...,2n + 2}
We obtain the total dominator coloring as follows: For $1 \leq a \leq n$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1 (mod\ 2) \\ i+2, & for\ b \equiv 0 (mod\ 2) \end{cases}$$

$$C(x_a) = \{n+3, n+4, n+5, ..., 2n+2\}$$

- Allocation of distinct colors to the vertices x_a makes it possible for the other vertices v_a and x_{ab} in every i - th copy to dominate the vertices x_a.

- Since it is a total dominator coloring, these vertices x_a in turn require atleast one distinctly colored vertex in every i - th copy to dominate. The remaining vertices can be colored using repeated colors if required.

- Usage of more number of colors than 2n+2, say if, $\chi_d^t(P_n \circledast P_m) \geq 2n+2$, will provide a maximum chromatic number. Using colors less than 2n+2, say if, $\chi_d^t(P_n \circledast P_m) \leq 2n+2$, will not satisfy the total dominator coloring property.

Hence we require 2n + 2 colors.

$$\chi_d^t(P_n \circledast P_m) = 2n + 2$$

Theorem 3.3 *For any integers $n \geq 2$, $m \geq 3$, the dominator chromatic number of duplication corresponding corona of path P_n with cycle C_m is*

$$\chi_d(P_n \circledast C_m) = \begin{cases} n+4, \; when\ m\ is\ odd \\ n+3, \; when\ m\ is\ even \end{cases}$$

proof Let P_n be the path graph with vertex set $V(P_n)$ and $\mathscr{D}P_n$ denote the duplication of the graph P_n. The vertices in the duplication graph are defined as

$$V(\mathscr{D}P_n) = \{v_a, x_a : 1 \leq a \leq n\}$$

Let us consider 'n' copies of cycle graph C_m, whose vertices are defined as,

$$V(C_m) = \{x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m\}$$

Let $'i'$ denote the respective copy of C_m, where $1 \leq i \leq n$, so that the vertex set of the duplication corresponding corona graph is given by,

$$V(P_n \circledast C_m) = \{v_a, x_a, x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m\}$$

The cardinality of the vertex set of the resultant graph is given by $\mid V(P_n \circledast C_m) \mid = n(m+2)$. The maximum and minimum degree of the graph are $\Delta\ (P_n \circledast C_m) = m+2$ and $\delta\ (P_n \circledast C_m) = 3$ respectively. Let the coloring be represented as $C : V(P_n \circledast C_m) \rightarrow \{1, 2, ..., \chi_d(P_n \circledast C_m)\}$
We obtain the dominator coloring in two cases as follows:
Case 1 : m is odd
For $1 \leq a \leq n$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2),\ b \neq m \\ 3, & for\ b \equiv 0(mod\ 2) \\ 4, & for\ b = m \end{cases}$$

$$C(x_a) = a + 4$$

Case 2 : m is even
For $1 \leq a \leq n$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \end{cases}$$

$$C(x_a) = a + 3$$

- Allocation of distinct colors to the vertices x_a makes it possible for the other vertices v_a and x_{ab} in every i-th copy to dominate the vertices x_a. These vertices x_a are self - dominating.

- Using more number of colors than mentioned, say if, $\chi_d(P_n \circledast C_m) \geq n+4$ when m is odd and if, $\chi_d(P_n \circledast C_m) \geq n+3$ when m is even, we will arrive at a maximum coloring. Using colors less than these, say if, $\chi_d(P_n \circledast C_m) \leq n+4$ when m is odd and if, $\chi_d(P_n \circledast C_m) \leq n+3$ when m is even, will not satisfy the dominator coloring property.

Hence we require n + 4 colors when m is odd and n + 3 colors when m is even.

$$\chi_d(P_n \circledast C_m) = \begin{cases} n+4, \text{ when m is odd} \\ n+3, \text{ when m is even} \end{cases}$$

Theorem 3.4 *For any integers $n \geq 2$, $m \geq 3$, the total dominator chromatic number of duplication corresponding corona of path P_n with cycle C_m is*

$$\chi_d^t(P_n \circledast C_m) = \begin{cases} 2n+3, \ when\ m\ is\ odd \\ 2n+2, \ when\ m\ is\ even \end{cases}$$

proof Let P_n be the path graph with vertex set $V(P_n)$ and $\mathscr{D}P_n$ denote the duplication of the graph P_n. The vertices in the duplication graph are defined as

$$V(\mathscr{D}P_n) = \{v_a, x_a : 1 \leq a \leq n\}$$

Let us consider 'n' copies of C_m, whose vertices are defined as,

$$V(C_m) = \{x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m\}$$

Let $'i'$ denote respective copy of C_m, where $1 \leq i \leq n$, so that the vertex set of the duplication corresponding corona graph is given by,

$$V(P_n \circledast C_m) = \{v_a, x_a, x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m\}$$

The cardinality of the vertex set and the degrees are followed from the previous theorem.

Let the coloring be represented as $C : V(P_n \circledast C_m) \rightarrow \{1, 2, ..., \chi_d^t(P_n \circledast C_m)\}$ We obtain the total dominator coloring in two cases as follows:
Case 1 : m is odd
For $1 \leq a \leq n$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \\ \{4, 5, 6, ..., n+3\} & for\ b = m \end{cases}$$

$$C(x_a) = \{n+4, n+5, ..., 2n+3\}$$

Case 2 : m is even
For $1 \leq a \leq n$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ i+2, & for\ b \equiv 0(mod\ 2) \end{cases}$$

$$C(x_a) = \{n+3, n+4, ..., 2n+2\}$$

- Allocation of distinct colors to the vertices x_a makes it possible for other vertices in every i - th copy to dominate the vertices x_a.
- Since it is a total dominator coloring, these vertices x_a in turn require a distinct color in every i - th copy to dominate.The remaining vertices can be colored using repeated colors if required.
- Using more number of colors than mentioned, say if, $\chi_d(P_n \circledast C_m) \geq 2n+3$ when m is odd and if, $\chi_d(P_n \circledast C_m) \geq 2n+2$ when m is even, we will arrive at a maximum chromatic number. Using colors less than these colors, say if, $\chi_d(P_n \circledast C_m) \leq 2n+3$ when m is odd and if, $\chi_d(P_n \circledast C_m) \leq 2n+2$ when m is even, will not satisfy the total dominator coloring property.

Hence we require 2n + 3 colors when m is odd and 2n + 2 colors when m is even.

$$\chi_d^t(P_n \circledast C_m) = \begin{cases} 2n+3, \text{ when m is odd} \\ 2n+2, \text{ when m is even} \end{cases}$$

Theorem 3.5 *For any integers $n \geq 3$, $m \geq 2$, the dominator chromatic number of duplication corresponding corona of cycle C_n with path P_m is*

$$\chi_d(C_n \circledast P_m) = n+3$$

The result and the proof is followed from Theorem 3.1.

Theorem 3.6 *For any integer n, ≥ 3, $m \geq 2$, The total dominator chromatic number of duplication corresponding corona of cycle C_n with path P_m is*

$$\chi_d^t(C_n \circledast P_m) = 2n + 2$$

The result and proof is in accordance with Theorem 3.2.

Theorem 3.7 *For any integer n,m ≥ 3, The dominator chromatic number of duplication corresponding corona of cycle C_n with cycle C_m is*

$$\chi_d(C_n \circledast C_m) = \begin{cases} n+4, \text{ when } m \text{ is odd} \\ n+3, \text{ when } m \text{ is even} \end{cases}$$

The result and proof follows from Theorem 3.3.

Theorem 3.8 *For any integer n,m ≥ 3, The total dominator chromatic number of duplication corresponding corona of cycle C_n with cycle C_m is*

$$\chi_d^t(C_n \circledast C_m) = \begin{cases} 2n+3, \text{ when } m \text{ is odd} \\ 2n+2, \text{ when } m \text{ is even} \end{cases}$$

The result and proof is in accordance with Theorem 3.4.

Theorem 3.9 *For any integers $n \geq 2$, $m \geq 3$, The dominator chromatic number of duplication corresponding corona of path P_n with wheel $W_{1,m}$ is*

$$\chi_d(P_n \circledast W_{1,m}) = \begin{cases} n+5, \text{ when } m \text{ is odd} \\ n+4, \text{ when } m \text{ is even} \end{cases}$$

proof

Let P_n be the path graph with vertex set $V(P_n)$ and $\mathscr{D}P_n$ denote the duplication of the graph P_n. The vertices in the duplication graph are defined as

$$V(\mathscr{D}P_n) = \{v_a, x_a : 1 \leq a \leq n\}$$

Let us consider 'n' copies of wheel graph $W_{1,m}$, whose vertices are defined as,

$$V(W_{1,m}) = \{x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m+1\}$$

Let $'i'$ denote the respective copy of $W_{1,m}$, where $1 \leq i \leq n$, so that the vertex set of the duplication corresponding corona graph is given by,

$$V(P_n \circledast W_{1,m}) = \{v_a, x_a, x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m+1\}$$

The cardinality of the vertex set of the resultant graph is given by $\mid V(P_n \circledast W_{1,m}) \mid = n(m+3)$. The maximum and minimum degree of the graph are $\Delta\ (P_n \circledast W_{1,m}) = m+2$ and $\delta\ (P_n \circledast W_{1,m}) = 3$ respectively.

Let the coloring be represented as $C : V(P_n \circledast W_{1,m}) \rightarrow \{1, 2, ..., \chi_d(P_n \circledast W_{1,m})\}$

We obtain the dominator coloring in two cases as follows:

Case 1: m is odd

For $1 \leq a \leq n$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \\ 4, & for\ b = m \\ 5, & for\ b = m+1 \end{cases}$$

$$C(x_a) = a + 5$$

Case 2: m is even

For $1 \leq a \leq n$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \\ 4, & for\ b = m+1 \end{cases}$$

$$C(x_a) = a + 4$$

- Allocation of distinct colors to the vertices x_a makes it possible for other vertices in every i - th copy to dominate the vertices x_a. These vertices x_a are self dominating.

- Usage of more number of colors than mentioned, say if, $\chi_d(P_n \circledast W_{1,m}) \geq n+5$ when m is odd and if, $\chi_d(P_n \circledast W_{1,m}) \geq n+4$ when m is even, will produce a maximum chromatic number. Using colors less than these colors, say if, $\chi_d(P_n \circledast W_{1,m}) \leq n+5$ when m is odd and if, $\chi_d(P_n \circledast W_{1,m}) \leq n+4$ when m is even, will not satisfy the dominator coloring property.

- Hence we require n + 5 colors when m is odd and n + 4 colors when m is even.

$$\chi_d(P_n \circledast W_{1,m}) = \begin{cases} n+5, \text{ for m is odd} \\ n+4, \text{ for m is even} \end{cases}$$

Theorem 3.10 *For any integers $n \geq 2$, $m \geq 3$, the total dominator chromatic number of duplication corresponding corona of path P_n with wheel $W_{1,m}$ is*

$$\chi_d^t(P_n \circledast W_{1,m}) \begin{cases} 2n+4, & \text{when } m \text{ is odd} \\ 2n+3, & \text{when } m \text{ is even} \end{cases}$$

proof Let P_n be the path graph with vertex set $V(P_n)$ and $\mathscr{D}P_n$ denote the duplication of the graph P_n. The vertices in the duplication graph are defined as

$$V(\mathscr{D}P_n) = \{v_a, x_a : 1 \leq a \leq n\}$$

Let us consider 'n' copies of wheel graph $W_{1,m}$, whose vertices are defined as,

$$V(W_{1,m}) = \{x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m+1\}$$

Let $'i'$ denote respective copy of $W_{1,m}$, where $1 \leq i \leq n$, so that the vertex set of the duplication corresponding corona graph is given by,

$$V(P_n \circledast W_{1,m}) = \{v_a, x_a, x_{ab} : 1 \leq a \leq n; 1 \leq b \leq m+1\}$$

The cardinality of the vertex set of the resultant graph is given by $| V(P_n \circledast W_{1,m}) |= n(m+3)$. The maximum and minimum degree of the graph are $\Delta\ (P_n \circledast W_{1,m}) = m+2$ and $\delta\ (P_n \circledast W_{1,m}) = 3$ respectively.

Let the coloring be represented as $C : V(P_n \circledast W_{1,m}) \to \{1, 2, ..., \chi_d^t(P_n \circledast W_{1,m})\}$

We obtain the total dominator coloring in two cases as follows:

Case 1: m is odd

For $1 \leq a \leq n$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \\ 4, & for\ b = m \\ \{5, 6, 7, ..., n+4\} & for\ b = m+1 \end{cases}$$

$$C(x_a) = \{n+5, n+6, ..., 2n+4\}$$

Case 2: m is even

For $1 \leq a \leq n$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \\ \{4, 5, 6, ..., n+3\} & for\ b = m+1 \end{cases}$$

$$C(x_a) = \{n+4, n+5, ..., 2n+3\}$$

- Allocation of distinct colors to the vertices x_a makes it possible for other vertices in every i - th copy to dominate the vertices x_a.
- Since it is a total dominator coloring, these vertices x_a in turn require a distinctly colored vertex in every i - th copy to dominate.The remaining vertices can be colored using repeated colors if required.
- Usage of more number of colors than mentioned, say if, $\chi_d(P_n \circledast W_{1,m}) \geq 2n+4$ when m is odd and if, $\chi_d(P_n \circledast W_{1,m}) \geq 2n+3$ when m is even, will produce a maximum chromatic number. Using colors less than these, say if, $\chi_d(P_n \circledast W_{1,m}) \leq 2n+4$ when m is odd and if, $\chi_d(P_n \circledast W_{1,m}) \leq 2n+3$ when m is even, will not satisfy the total dominator coloring property.
- Hence we require 2n + 4 colors when m is odd and 2n + 3 colors when m is even.

$$\chi_d^t(P_n \circledast W_{1,m}) = \begin{cases} 2n+4, \text{ for m is odd} \\ 2n+3, \text{ for m is even} \end{cases}$$

Theorem 3.11 *For any integers $n \geq 3$, $m \geq 2$, the dominator chromatic number of duplication corresponding corona of wheel $W_{1,n}$ with path P_m is*

$$\chi_d(W_{1,n} \circledast P_m) = n + 4$$

proof

Let $W_{1,n}$ be the wheel graph with vertex set $V(W_{1,n})$ and $\mathscr{D}W_{1,n}$ denote the duplication of the graph $W_{1,n}$.The vertices in the duplication graph are defined as

$$V(\mathscr{D}W_{1,n}) = \{v_a, x_a : 1 \leq a \leq n+1\}$$

Let us consider $'n+1'$ copies of path graph P_m, whose vertices are defined as,

$$V(P_m) = \{x_{ab} : 1 \leq a \leq n+1; 1 \leq b \leq m\}$$

Let $'i'$ denote respective copy of P_m, where $1 \leq i \leq n+1$, so that the vertex set of the duplication corresponding corona graph is given by,

$$V(W_{1,n} \circledast P_m) = \{v_a, x_a, x_{ab} : 1 \leq a \leq n+1; 1 \leq b \leq m\}$$

The cardinality of the vertex set of the resultant graph is given by $\mid V((W_{1,n} \circledast P_m) \mid= mn + 2n + m + 2$. The maximum and minimum degree of the graph are $\Delta\ (W_{1,n} \circledast P_m) = m + 3$ and $\delta\ (W_{1,n} \circledast P_m) = 3$ respectively.

Let the coloring be represented as $C : V(W_{1,n} \circledast P_m) \to \{1, 2, ..., n+4\}$ We obtain the dominator coloring as follows: For $1 \leq a \leq n+1$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \end{cases}$$

$$C(x_a) = a + 3$$

- Allocation of distinct colors to the vertices x_a makes it possible for other vertices in every i - th copy to dominate the vertices x_a. These vertices x_a are self - dominating.
- Usage of more number of colors than n+4, say if, $\chi_d(W_{1,n} \circledast P_m) \geq n+4$, will provide a maximum chromatic number. Using colors less than n+4, say if, $\chi_d(W_{1,n} \circledast P_m) \leq n+4$, will not satisfy the dominator coloring property.

Hence we require n + 4 colors.

$$\chi_d(W_{1,n} \circledast P_m) = n + 4$$

.

Theorem 3.12 *For any integers* $n \geq 3$*,* $m \geq 2$*, the total dominator chromatic number of duplication corresponding corona of wheel* $W_{1,n}$ *with path* P_m *is*

$$\chi_d^t(W_{1,n} \circledast P_m) = 2n + 4$$

proof

Let $W_{1,n}$ be the wheel graph with vertex set $V(W_{1,n})$ and $\mathscr{D}W_{1,n}$ denote the duplication of the graph $W_{1,n}$. The vertices in the duplication graph are defined as

$$V(\mathscr{D}W_{1,n}) = \{v_a, x_a : 1 \leq a \leq n+1\}$$

Let us consider $'n+1'$ copies of path graph P_m, whose vertices are defined as,

$$V(P_m) = \{x_{ab} : 1 \leq a \leq n+1; 1 \leq b \leq m\}$$

Let $'i'$ denote respective copy of P_m, where $1 \leq i \leq n+1$, so that the vertex set of the duplication corresponding corona graph is given by,

$$V(W_{1,n} \circledast P_m) = \{v_a, x_a, x_{ab} : 1 \leq a \leq n+1; 1 \leq b \leq m\}$$

The cardinality of the vertex set of the resultant graph is given by $\mid V((W_{1,n} \circledast P_m) \mid= mn + 2n + m + 2$. The maximum and minimum degree of the graph are $\Delta\ (W_{1,n} \circledast P_m) = m + 3$ and $\delta\ (W_{1,n} \circledast P_m) = 3$ respectively.

Let the coloring be represented as $C : V(W_{1,n} \circledast P_m) \rightarrow \{1, 2, ..., 2n + 4\}$ We obtain the total dominator coloring as follows: For $1 \leq a \leq n + 1$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ \{3,4,5,...,n+3\} & for\ b \equiv 0(mod\ 2) \end{cases}$$

$$C(x_a) = \{n+4, n+5, ..., 2n+4\}$$

- Allocation of distinct colors to the vertices x_a makes it possible for other vertices in every i - th copy to dominate the vertices x_a.
- These vertices x_a in turn require a distinct color in every i - th copy to dominate. The remaining vertices can be colored using repeated colors if required.
- Usage of more number of colors than 2n+4, say if, $\chi_d(W_{1,n} \circledast P_m) \geq 2n+4$, will provide a maximum chromatic number. Using colors less than 2n+4, say if, $\chi_d(W_{1,n} \circledast P_m) \leq 2n+4$, will not satisfy the total dominator coloring property.

Hence we require 2n + 4 colors.

$$\chi_d^t(W_{1,n} \circledast P_m) = 2n + 4$$

.

Theorem 3.13 *For any integers $n, m \geq 3$, the dominator chromatic number of duplication corresponding corona of wheel $W_{1,n}$ with wheel $W_{1,m}$ is*

$$\chi_d(W_{1,n} \circledast W_{1,m}) = \begin{cases} n+6, & when\ m\ is\ odd \\ n+5, & when\ m\ is\ even \end{cases}$$

proof Let $W_{1,n}$ be the wheel graph with vertex set $V(W_{1,n})$ and $\mathscr{D}W_{1,n}$ denote the duplication of the graph $W_{1,n}$. The vertices in the duplication graph are defined as

$$V(\mathscr{D}W_{1,n}) = \{v_a, x_a : 1 \leq a \leq n+1\}$$

Let us consider $'n+1'$ copies of wheel graph $W_{1,m}$, whose vertices are defined as,

$$V(W_{1,m}) = \{x_{ab} : 1 \leq a \leq n+1; 1 \leq b \leq m+1\}$$

Let $'i'$ denote respective copy of $V(W_{1,m})$, where $1 \leq i \leq n+1$, so that the vertex set of the duplication corresponding corona graph is given by,

$$V(W_{1,n} \circledast W_{1,m}) = \{v_a, x_a, x_{ab} : 1 \leq a \leq n+1; 1 \leq b \leq m+1\}$$

The cardinality of the vertex set of the resultant graph is given by $| V(W_{1,n} \circledast W_{1,m}) | = mn + 3n + m + 3$. The maximum and minimum degree of the graph are $\Delta\ (W_{1,n} \circledast W_{1,m}) = m+3$ and $\delta\ (W_{1,n} \circledast W_{1,m}) = 4$ respectively.

Let the coloring be represented as $C : V(W_{1,n} \circledast W_{1,m}) \rightarrow \{1, 2, ..., \chi_d(W_{1,n} \circledast W_{1,m})\}$
We obtain the dominator coloring in two cases as follows:
Case 1: m is odd
For $1 \leq a \leq n+1$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \\ 4, & for\ b = m \\ 5, & for\ b = m+1 \end{cases}$$

$$C(x_a) = a + 5$$

Case 2: m is even
For $1 \leq a \leq n+1$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \\ 4, & for\ b = m+1 \end{cases}$$

$$C(x_a) = a + 4$$

- Allocation of distinct colors to the vertices x_a makes it possible for other vertices in every i - th copy to dominate the vertices x_a. These vertices x_a are self - dominating.

- Usage of more number of colors than said, say if, $\chi_d(W_{1,n} \circledast W_{1,m}) \geq n+6$ when m is odd and if, $\chi_d(W_{1,n} \circledast W_{1,m}) \geq n+5$ when m is even, will provide a maximum chromatic number. Using colors less than these, say if, $\chi_d(W_{1,n} \circledast W_{1,m}) \leq n+6$ when m is odd and if, $\chi_d(W_{1,n} \circledast W_{1,m}) \leq n+5$ when m is even, will not satisfy the dominator coloring property.

- Hence we require n + 6 colors when m is odd and n + 5 colors when m is even.

$$\chi_d(W_{1,n} \circledast W_{1,m}) = \begin{cases} n+6, & \text{for m is odd} \\ n+5, & \text{for m is even} \end{cases}$$

Theorem 3.14 *For any integers $n, m \geq 3$, the total dominator chromatic number of duplication corresponding corona of wheel $W_{1,n}$ with wheel $W_{1,m}$ is*

$$\chi_d^t(W_{1,n} \circledast W_{1,m}) = \begin{cases} 2n+6, \text{ for } m \text{ is odd} \\ 2n+5, \text{ for } m \text{ is even} \end{cases}$$

proof Let $W_{1,n}$ be the wheel graph with vertex set $V(W_{1,n})$ and $\mathscr{D}W_{1,n}$ denote the duplication of the graph $W_{1,n}$. The vertices in the duplication graph are defined as

$$V(\mathscr{D}W_{1,n}) = \{v_a, x_a : 1 \leq a \leq n+1\}$$

Let us consider $'n'$ copies of wheel graph $W_{1,m}$, whose vertices are defined as,

$$V(W_{1,m}) = \{x_{ab} : 1 \leq a \leq n+1; 1 \leq b \leq m+1\}$$

Let $'i'$ denote respective copy of $V(W_{1,m})$, where $1 \leq i \leq n+1$, so that the vertex set of the duplication corresponding corona graph is given by,

$$V(W_{1,n} \circledast W_{1,m}) = \{v_a, x_a, x_{ab} : 1 \leq a \leq n+1; 1 \leq b \leq m+1\}$$

The cardinality of the vertex set of the resultant graph is given by $| V(W_{1,n} \circledast W_{1,m}) | = mn + 3n + m + 3$. The maximum and minimum degree of the graph are Δ $(W_{1,n} \circledast W_{1,m}) = m+3$ and δ $(W_{1,n} \circledast W_{1,m}) = 4$ respectively.

Let the coloring be represented as $C : V(W_{1,n} \circledast W_{1,m}) \to \{1, 2, ..., \chi_d^t(W_{1,n} \circledast W_{1,m})\}$

We obtain the dominator coloring in two cases as follows:

Case 1: m is odd

For $1 \leq a \leq n+1$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \\ 4, & for\ b = m \\ \{5, 6, 7, ..., n+5\} & for\ b = m+1 \end{cases}$$

$$C(x_a) = \{n+6, n+7, ..., 2n+6\}$$

Case 2: m is even

For $1 \leq a \leq n+1$, map the coloring function as

$$C(v_a) = 1$$

$$C(x_{ab}) = \begin{cases} 2, & for\ b \equiv 1(mod\ 2) \\ 3, & for\ b \equiv 0(mod\ 2) \\ \{4, 5, 6, ..., 2n+5\} & for\ b = m+1 \end{cases}$$

$$C(x_a) = \{n+5, n+6, ..., 2n+5\}$$

- Allocation of distinct colors to the vertices x_a makes it possible for other vertices in every i - th copy to dominate the vertices x_a.
- These vertices x_a in turn require a distinct color in every i - th copy to dominate.The remaining vertices can be colored using repeated colors if required.
- Usage of more number of colors than said, say if, $\chi_d(W_{1,n} \circledast W_{1,m}) \geq 2n+6$ when m is odd and if, $\chi_d(W_{1,n} \circledast W_{1,m}) \geq 2n+5$ when m is even, will provide a maximum chromatic number. Using colors less than these, say if, $\chi_d(W_{1,n} \circledast W_{1,m}) \leq 2n+6$ when m is odd and if, $\chi_d(W_{1,n} \circledast W_{1,m}) \leq 2n+5$ when m is even, will not satisfy the total dominator coloring property.
- Hence we require 2n + 6 colors when m is odd and 2n + 5 colors when m is even.

$$\chi_d^t(W_{1,n} \circledast W_{1,m}) = \begin{cases} 2n+6, \text{ for m is odd} \\ 2n+5, \text{ for m is even} \end{cases}$$

4 Conclusion

In this paper, we have applied the dominator and total dominator coloring and have found the dominator and total dominator chromatic number of duplication corresponding corona of path, cycle and wheel graphs. This work shall be continued for some other types of graphs.

5 Open Problem

Problem 1: Characterize graphs $\mathscr{G}$ for which

$$\chi_d(P_n \circledast \mathscr{G}) = \chi_d^t(P_n \circledast \mathscr{G})$$

.

Problem 2: Characterize graphs $\mathscr{G}$ for which

$$\chi_d(C_n \circledast \mathscr{G}) = \chi_d^t(C_n \circledast \mathscr{G})$$

.

Problem 3: Provide a Generalization for any $\mathscr{G}$,
what is

$$\chi_d(P_n \circledast \mathscr{G}) \text{ and } \chi_d(C_n \circledast \mathscr{G})$$

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