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Neutrosophic M- semigroup

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Abstract

The puropse of this paper is to generalize the structure of fuzzy M semigroup in neutrosophic setting. As a consequence the notion of neutrosophic M- semigroup is introduced with interesting results.

Keywords: *M-semigroup, fuzzy M- semigroup, antifuzzy M-semigroup, neutrosophic M-semigroup.*

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1 Introduction

Lakshmanan [1] introduced the concept of M-semigroups, which brought new insights into the study of semigroups. An M-semigroup is a specialized type of semigroup with additional properties beyond the basic associative operation. While a traditional semigroup involves an associative operation on its elements, an M-semigroup incorporates specific rules or characteristics that define its behavior. These properties can vary based on the particular type of M-semigroup being studied, providing a more nuanced understanding of how these algebraic structures function and interact. His research focused on understanding the structure of subsemigroups, examining how they behave and interact within a larger system. He also explored the role of ideals, which are specific subsets that help define the overall organization of an M-semigroup.

Through his analysis, Lakshmanan provided a better understanding of how these mathematical structures work.

He explored various aspects of M-semigroups, focusing on how subsemigroups function within this structure. His research covered the characteristics and behaviors of these smaller semigroups, which are subsets of M-semigroups [2]. He also looked into the structure of ideals in M-semigroups [3], examining how these special subsets influence the overall organization and properties of the semigroup. Additionally, he studied the role of dominators within M-semigroups [4], which are elements that have particular significance in defining and understanding the semigroup's behavior. His work provided a thorough analysis of these components, contributing to a deeper understanding of M-semigroups.

Narayanan and Meenakshi [5] then extended the theory of M-semigroups into fuzzy mathematics, which deals with reasoning that allows for uncertainty and approximation. This extension showed that M-semigroups could be applied to areas where traditional logic does not fully apply. Their work demonstrated how flexible and adaptable the concept of M-semigroups could be.

Later, Vijayabalaji and Sivaramakrishnan [7] took the idea further by exploring M-semigroups in the anti-fuzzy context. This new approach adapted the theory to a different type of logical framework, showing that M-semigroups could be used in even more diverse mathematical settings. Their research added another layer of depth to the theory.

Following Lakshmanan's work, Vijayabalaji and Shakila focused on particular types of M-semigroups, such as 0-simple and soft M-semigroups [8]. These specific forms have unique features that make them important for certain mathematical problems. By studying these forms, Vijayabalaji and Shakila expanded the theory, making it more useful for practical applications.

Overall, these developments in M-semigroup theory, from Lakshmanan's foundational work to the extensions by others, have broadened its applications and demonstrated its versatility. Each researcher contributed to making M-semigroups a more powerful tool in mathematical studies, showing their potential in various fields.

The concept of a neutrosophic set was introduced by Florentin Smarandache [6]. Neutrosophic sets extend traditional fuzzy sets by incorporating three degrees of membership: truth, indeterminacy, and falsity. This allows for a more flexible representation of uncertainty and partial information. Smarandache's introduction of neutrosophic sets aimed to address limitations in classical fuzzy logic, providing a broader framework for analyzing complex and ambiguous situations.

Wang et al. [9] developed the single-valued neutrosophic set to extend the traditional fuzzy set theory. This new concept allows for more nuanced representations of uncertainty, as it includes three membership degrees: truth, indeterminacy, and falsity. Unlike fuzzy sets, which only consider the degree of truth, single-valued neutrosophic sets offer a more flexible approach by accounting for indeterminate and false information. This makes them particularly useful in complex decision-making and modeling scenarios where uncertainty plays a significant role. The introduction of this concept has expanded the possibilities for analyzing and interpreting data in various fields, including artificial intelligence, information systems, and decision sciences.

1.1 Scope and objectives of the present investigation

This research focuses on advancing the study of M-semigroups within the context of neutrosophic logic. It explores how M-semigroups can be adapted to neutrosophic settings, which are characterized by handling uncertainty and partial truth. One key finding is that when two neutrosophic M-semigroups are intersected, the result is another neutrosophic M-semigroup. This result is important because it shows that the intersection operation preserves the structure of the M-semigroup even in the neutrosophic environment.

Additionally, the research examines the behavior of the product of two neutrosophic M-semigroups. It investigates how the multiplication operation between these semigroups works and how it affects their properties within the neutrosophic framework. These findings contribute to a better understanding of how M-semigroups interact in a neutrosophic context and provide insights into their structural behavior under various operations.

2 Preliminaries

Let's revisit the concept of an M-semigroup and explore the recent developments in its structures.

Definition 2.1 [**1**] An M- semigroup Υ is a semigroup with two more conditions namely

(i) there exists atleast one left identity $e \in \Upsilon$ such that $ex = x$ for all $x \in \Upsilon$.

(ii) For every $x \in \Upsilon$, there exists a unique left identity e_x such that $xe_x = x$.

Definition 2.2 [**5**] Given an M-semigroup Υ, by a fuzzy M- semigroup U, we mean a function $U : \Upsilon \to [0, 1]$ such that

(i) $U(xy) \geq \min\{U(x), U(y)\}$

(ii) $U(e) = 1$ for every left identity $e \in \Upsilon$.

Definition 2.3 [**7**] Given an M-semigroup Υ, by an anti fuzzy M- semigroup G, we mean a function : $\Upsilon \to [0, 1]$ such that

(i) $G(xy) \leq \max\{U(x), U(y)\}$
(ii) $G(e) = 0$ for every left identity $e \in \Upsilon$.

Definition 2.4. [6] A neutrosophic set in an universe set U is defined by $A = \{(u, TA(u), IA(u), FA(u)) : u \in U\}$, where u being the generic element of U, TA being truth-membership function, IA being indeterminacymembership function and FA represents falsity-membership function.

3 Neutrosophic M-semigroup

Thoroughout the paper let e denotes the left identity in an M- semigroup Υ, Θ being right singular semigroup (that is for $x, y \in \Theta$, $xy = y$) ,κ is a semigroup with two sided identity (that is for $e \in \kappa$, $xe = ex = x$), $\star$ represents the t-norm and $\lhd$ denotes the t-co-norm.

Definition 3.1. Given an M-semigroup Υ with $x, y \in \Upsilon$, a neutrosophic M-semigroup (briefly NMSG) represented by $\aleph = \{(l, U(l), S(l), H(l))|l \in \Upsilon\}$ with
(i) $U(xy) \geq U(x) \star U(y)$
(ii) $U(e) = 1$
(iii) $S(xy) \geq S(x) \star S(y)$
(iv) $S(e) = 1$
(v) $H(xy) \leq H(x) \lhd H(y)$
(vi) $H(e) = 0$

where $U, S, H \to [0, 1]$ and $0 \leq U(l) + S(l) + H(l) \leq 3$ representing the true membership function, intermediate membership function and false membership function respectively.

Example 3.2. Consider an M-semigroup $\Upsilon = \{e, f, a, b\}$ with the following Cayley table.

	e	f	a	b
e	e	f	a	b
f	e	f	a	b
a	a	b	e	f
b	a	b	e	f

Define $U, S : \Upsilon \to [0, 1]$ by

$$U(l) = S(l) = \begin{cases} 1, & if\ l = e, f \\ t, & \text{otherwise}, 0 \leq t < 1. \end{cases}$$

Also define $H : \Upsilon \to [0, 1]$ by

$$H(l) = \begin{cases} 0, & if\ l = e, f \\ \alpha, & \text{otherwise}, 0 < \alpha \leq 1. \end{cases}$$

Then $\aleph$ is a NMSG.

Theorem 3.3. Given two neutrosophic M- semigroups (briefly NMSGS) $\aleph_1 = \{U_1, S_1, H_1\}$ and $\aleph_2 = \{U_2, S_2, H_2\}$, then $\aleph_1 \cap \aleph_2$ is NMSG with
(a) $(U_1 \cap U_2)(x) = \min\{U_1(x), U_2(x)\}$
(b) $(S_1 \cap S_2)(x) = \min\{S_1(x), S_2(x)\}$
(c) $(H_1 \cup H_2)(x) = \max\{U_1(x), U_2(x)\}$
Proof. For all $x, y \in \Upsilon$
$(U_1 \cap U_2)(xy)$
$= \min\ \{U_1(xy), U_2(xy)\}$
$\geq \min\ \{\min\{U_1(x), U_1(y)\}, \min\{U_2(x), U_2(y)\}\}$
$= \min\ \{\min\{U_1(x), U_2(x)\}, \min\{U_1(y), U_2(y)\}\}$
$= \min\ \{(U_1 \cap U_2)(x), (U_1 \cap U_2)(y)\}$
$\Rightarrow (U_1 \cap U_2)(xy) \geq \min\ \{(U_1 \cap U_2)(x), (U_1 \cap U_2)(y)\}.$
Also
$(U_1 \cap U_2)(e)$
$=\min\{U_1(e), U_2(e)\}$
$=\min\{1, 1\}$
$=1$
$\Rightarrow (U_1 \cap U_2)(e) = 1.$
The same argument can be used for $S_1 \cap S_2$.
Now
$(H_1 \cup H_2)(xy)$
$= \max\ \{H_1(xy), H_2(xy)\}$
$\leq \max\ \{\max\{H_1(x), H_1(y)\}, \max\{H_2(x), H_2(y)\}\}$
$= \max\ \{\max\{H_1(x), H_2(x)\}, \max\{H_1(y), H_2(y)\}\}$
$= \max\ \{(H_1 \cap H_2)(x), (H_1 \cap H_2)(y)\}$
$\Rightarrow (H_1 \cap H_2)(xy) \geq \min\ \{(H_1 \cap H_2)(x), (H_1 \cap H_2)(y)\}.$
Also
$(H_1 \cup H_2)(e)$
$=\max\{H_1(e), H_2(e)\}$
$=\max\{0, 0\}$
$=0.$

Theorem 3.4. Given two NMSGS, $\aleph_1 = \{U_1, S_1, H_1\}$ (of Θ) and $\aleph_2 = \{U_2, S_2, H_2\}$ (of κ) with $e, f \in \Theta$ and $x_1, x_2 \in \kappa$, then
(a) $\min\{\min\{U_1(e), U_1(f)\}, \min\{U_2(x_1), U_2(x_2)\}\}$
$\quad = \min\{\min\{U_1(e), U_2(x_1)\}, \min\{U_1(f), U_2(x_2)\}\}$
(b) $\min\{\min\{S_1(e), S_1(f)\}, \min\{S_2(x_1), S_2(x_2)\}\}$
$\quad = \min\{\min\{S_1(e), S_2(x_1)\}, \min\{S_1(f), S_2(x_2)\}\}$
(c) $\max\{\max\{H_1(e), H_1(f)\}, \max\{H_2(x_1), H_2(x_2)\}\}$

$= \max\{\max\{H_1(e), H_2(x_1)\}, \max\{H_1(f), H_2(x_2)\}\}$.

Proof.

(a) $\min\{\min\{U_1(e), U_1(f)\}, \min\{U_2(x_1), U_2(x_2)\}\}$
$=\min\{U_1(e), \min\{U_1(f), \min\{U_2(x_1), U_2(x_2)\}\}$
$=\min\{U_1(e), \min\{\min\{U_1(f), U_2(x_1), U_2(x_2)\}\}$
$=\min\{U_1(e), \min\{\min\{U_2(x_1), U_1(f), U_2(x_2)\}\}$
$=\min\{\min\{U_1(e), U_2(x_1)\}, \min\{U_1(f), U_2(x_2)\}\}$

(b) $\min\{\min\{S_1(e), S_1(f)\}, \min\{S_2(x_1), S_2(x_2)\}\}$
$=\min\{S_1(e), \min\{S_1(f), \min\{S_2(x_1), S_2(x_2)\}\}$
$=\min\{S_1(e), \min\{\min\{S_1(f), S_2(x_1), S_2(x_2)\}\}$
$=\min\{S_1(e), \min\{\min\{S_2(x_1), S_1(f), S_2(x_2)\}\}$
$=\min\{\min\{S_1(e), S_2(x_1)\}, \min\{S_1(f), S_2(x_2)\}\}$

(c) $\max\{\max\{H_1(e), H_1(f)\}, \max\{H_2(x_1), H_2(x_2)\}\}$
$=\max\{H_1(e), \max\{H_1(f), \max\{H_2(x_1), H_2(x_2)\}\}$
$=\max\{H_1(e), \max\{\max\{H_1(f), H_2(x_1), H_2(x_2)\}\}$
$=\max\{H_1(e), \max\{\max\{H_2(x_1), H_1(f), H_2(x_2)\}\}$
$=\max\{\max\{H_1(e), H_2(x_1)\}, \max\{H_1(f), S_2(x_2)\}\}$.

Theorem 3.5. Given two NMSGS, $\aleph_1 = \{U_1, S_1, H_1\}$ (of Θ) and $\aleph_2 = \{U_2, S_2, H_2\}$ (of κ) with $e, f \in \Theta$ and $x_1, x_2 \in \kappa$, then $\aleph_1 \times \aleph_2 = \{U_1 \times U_2, S_1 \times S_2, H_1 \times H_2\}$ is a NSMSG with
(a) $U(x,y)$= $(U_1 \times U_2)(x,y)$= $\min\{U_1(x), U_2(y)\}$
(b) $S(x,y)$= $(S_1 \times S_2)(x,y)$= $\min\{S_1(x), S_2(y)\}$
(c) $H(x,y)$= $(H_1 \times H_2)(x,y)$= $\max\{H_1(x), H_2(y)\}$, $\star$= min and $\lhd$= max.

Proof.

Choose $x = (e, x_1)$, $y = (f, x_2)$ in $M \cong \Theta \times \kappa$, with $e, f \in \Theta$ and $x_1, x_2 \in \kappa$.

Now

(i) $U(xy)$
$=U(ef, x_1x_2)$
$=(U_1 \times U_2)(ef, x_1x_2)$
$=\min\{U_1(ef), U_2(x_1x_2)\}$.

By Definition 3.1, we have
$U(xy) \geq \min\{\min(U_1(e), U_1(f)), \min(U_2(x_1), U_2(x_2)\}$.
By Theorem 3.4.,
$U(xy)$
$\geq \min\{\min(U_1(e), U_2(x_1)), \min(U_1(f), U_2(x_2)\}$
$=\min\{(U_1 \times U_2)(e, x_1), (U_1 \times U_2)(f, x_2)\}$
$=\min\{U(x), U(y)\}$.
Also

(ii) $U(e_1, e)$

$=(U_1 \times U_2)(e_1, e)$,
where $e_1 \in \Theta$ and e is a two sided identity of κ.
$U(e_1, e)$
$=\min(U_1(e_1), U_2(e))$
$=\min\ (1, 1)$
$=1$.
In a similar way we can prove the conditions (iii)- (vi), using (a) and (b) and so $\aleph_1 \times \aleph_2$ is a NSMSG.

4 Conclusion

In conclusion, this research shows that M-semigroups can be effectively adapted to neutrosophic logic. It is demonstrated that the intersection of two neutrosophic M-semigroups results in another neutrosophic M-semigroup, preserving the original structure within the neutrosophic framework.

5 Open Problem

Our research suggests the following open problems:

1. Can we construct a neutrosophic cubic M-semigroup.

2. Is it possible to extend the concept of neutrosophic M-semigroups to neutrosophic soft M-semigroups.

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