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A study on intuitionistic fuzzy soft expert groups

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Abstract

Researcher's enthusiasm for developing various uncertainty theories is quite significant. Intuitionistic fuzzy soft sets are one of the most current uncertainty tools that performs admirably. The fundamental flaw in this theory is the absence of specialists, in fact that it has only one expert. To address this issue, the concept of IFSE-set theory has been developed recently. This structure is unique in the manner that it provides answers from multiple experts. This thought sparked the creation of the current research. On an intuitionistic fuzzy soft expert set, we focus our notion towards the development of algebraic structures. As a result, we constructed an intuitionistic fuzzy soft expert group (IFSE-group) for this project. We investigate the relationship between the IFSE-subgroup's identity (absolute, central, commutator) and the intuitionistic fuzzy soft expert group's identity (absolute, central, commutator). The article concludes with an overview of IFSE-homomorphism and provide some results on their subgroups.

Keywords: ***FSE-set, IFSE-set, IFSE-group, IFSE-subgroup, IFSE-homomorphism.***

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1 Introduction

The notion of soft sets as a new mathematical tool for dealing with uncertainty was introduced by Molodtsov [11]. The notion of restricted symmetric difference of soft sets are investigated by Sezgin and Atagün [15] and extended the theoretical aspect of operations on soft sets. Molodtsov's book [12] demonstrates the majority of these applications. The fundamental characteristics of soft sets are described by Aktas and Çağman [1], which also draws comparisons between soft sets and their associated concepts, fuzzy sets and rough sets. Using Molodtsov's description of soft sets, they provide a definition of soft group and derived its fundamental features. Kamaci [8] developed new operations in N-soft set and derived certain properties for algebraic structures in N-soft set. Vijayabalaji et al. [18] introduced the concept of push out and pull back on soft module homomorphism. Hybrid structures of rough modules and soft modules namely soft-rough modules over soft-rough rings and modified soft-rough modules over a modified soft-rough rings are also introduced.

A fuzzy set is a class of objects with a concept of membership grades that was first described by Zadeh [21]. Assigning membership grade ranging from zero to one is the natural phenomenon of membership function. Fuzzy sets extend the concepts of inclusion, complement, intersection, union, relation, convexity, etc., in membership version. Various studies on the generalization of fuzzy set were being done for the past firedecades. Xiao et al. [20] described the idea of fuzzy soft module and investigated some of its fundamental characteristics. The most up-to-date information on this topic was provided by Mordeson et al. [13]. Many researchers have incorporated the generalization of the concept of fuzzy set. The concept of intuitionistic fuzzy set was developed by Atanassov [4]. Intuitionistic fuzzy soft module was proposed by Gunduz and Bayramov [6], which extends the concept of modules with soft set theory and investigated their properties. The notion of soft expert set (SE-set) was introduced by Alkhazaleh and Salleh [3] which was more effective and efficient. They also explored the features of its basic operations: complement, intersection, union, AND and OR. Kalaiselvan and Vijayabalaji [7] generalized the notion of soft expert set to soft expert symmetric group. The application of a SES-group in MCDM situations is also presented in it. Alkhazaleh and Salleh [2] defined the concept of fuzzy soft expert set and discussed a mapping on fuzzy soft expert classes and its properties. The concept of intuitionistic fuzzy soft expert sets (IFSE-set) was introduced by Broumi and Smarandache [5], which combines intuitionistic fuzzy sets and soft expert sets. This concept is a generalization of FSE-set.

The goal of this study is to broaden the concept of groups in IFSE-set algebraic structures. IFSE-group, IFSE-subgroup and identity (absolute, central, commutator) IFSE-subgroup of IFSE-group are defined and the results are proven. We prove significant results such as: AND operator on conditionally IFSE-subgroup is an IFSE-subgroup also the product of two IFSE-groups is an IFSE-group if and only if one of them is a normal IFSE-group. The concept IFSE-homomorphism is constructed and useful parts such as Image and Kernel of an IFSE-homomorphism are being derived. Then, IFSE-monomorphism, IFSE-epimorphism are naturally defined and intriguing results are obtained.

2 Preliminaries

Throughout this paper, U be an universe, $\mathcal{Z} = E \times X \times O$ and $\mathcal{L} \subseteq \mathcal{Z}$, where E, X and $O = \{0 = \text{agree}, 1 = \text{disagree}\}$ be the collection of parameters, experts(agents) and opinions respectively. $P(U)$ denote the power set of U. Let G be fuzzy group, I^G denotes the collection of fuzzy subset of G, $I = [0, 1]$. e be an identity element in G. Γ be a homomorphism from G onto G'. $G_1 \leq_F G$ denotes that G_1 is a fuzzy subgroup in G, $N \trianglelefteq_F G$ denotes that N is a normal fuzzy subgroup in G. $(F_\mu, F_\nu, \mathcal{L})_U$ denotes that $(F_\mu, F_\nu, \mathcal{L})$ is an IFSE-set in U.

Definition 2.1. *[21] A fuzzy set A in X is characterized by $\mu_A(x)$ which associates with each points in X a real number in $[0, 1]$, where $\mu_A(x)$ is a membership function.*

Definition 2.2. *[14] μ is said to be fuzzy group of G, if for all $x_1, x_2 \in G$, $\mu_G(x_1x_2) \geq \min(\mu_G(x_1), \mu_G(x_2))$ and $\mu_G(x_1^{-1}) \geq \mu_G(x_1)$.*

Definition 2.3. *[11] Consider a nonempty set A, $A \subseteq E$. If $\aleph : A \to P(U)$ is a function, the the pair $(\aleph, A)$ is said to be soft set over U.*

Definition 2.4. *[3] Let $\mathcal{L} \subseteq \mathcal{Z}$, If $\aleph : \mathcal{L} \to P(U)$is a function, then the pair $(\aleph, \mathcal{L})_U$ is known as soft expert set (SE-set).*

Definition 2.5. *[2] Let $\mathcal{L} \subseteq \mathcal{Z}$. The pair $(\digamma_\mu, \mathcal{L})_U$ is known as fuzzy soft expert set (FSE-set), if $\digamma_\mu : \mathcal{L} \to I^U$ is a function. (i.e) a FSE-set $(\digamma_\mu, \mathcal{L})_U = \{(a, k_{\mu_{\digamma(a)}(k)})/k \in U, a \in \mathcal{L}\}$.*

Definition 2.6. *[4] Let U be an universe, let $A = \{(k, \mu_A(k), \nu_A(k)), k \in U\}$ be the set of objects with the condition $0 \leq \mu_A(k) + \nu_A(k) \leq 1$ is said to be intuitionistic fuzzy set, where the functions, $\mu_A(k), \nu_A(k) : U \to [0,1]$ respectively define the degree of membership, non-membership of the element $k \in X$ to the set A.*

Definition 2.7. *[5] Let U be an universe, $\mathcal{L} = E \times X \times O$, the object of the form $\{(x, \{r_{\mu_{\digamma(x)}(r), \nu_{\digamma(x)}(r)}, r \in \digamma(x)\}), x \in \mathcal{L}\}$ with $0 \leq \mu_{\digamma(x)}(r) + \nu_{\digamma(x)}(r) \leq 1$ is known as intuitionistic fuzzy soft expert set (IFSE-set) and is denoted by $(\digamma_\mu, \digamma_\nu, \mathcal{L})$. Where the functions, $\mu_{\digamma(x)}(r), \nu_{\digamma(x)}(r) : U \to [0,1]$ define respectively the degree of membership, non-membership of the element $r \in U$ to the set $\digamma(x)$.*

3 Intuitionistic fuzzy soft expert group

We now introduce our new interesting idea into an IFSE-set, namely intuitionistic fuzzy soft expert group (IFSE-group).

Definition 3.1. *A SE-set $(\aleph, \mathcal{L})_G$ is said to be soft expert group (SE-group), if for every $x \in \mathcal{L}$, $\aleph(x) \leq G$.*

Table 1: SE-set

$\aleph$	(1)	(1 2)	(1 3)	(2 3)	(1 2 3)	(1 3 2)
$(e_1, p_1, 1)$	1	1	0	0	0	0
$(e_1, p_1, 0)$	0	0	1	1	1	1
$(e_1, p_2, 1)$	0	1	1	1	0	0
$(e_1, p_2, 0)$	1	0	0	0	1	1
$(e_2, p_1, 1)$	1	0	0	1	1	1
$(e_2, p_1, 0)$	0	1	1	0	0	0
$(e_2, p_2, 1)$	1	0	0	0	1	1
$(e_2, p_2, 0)$	0	1	1	1	0	0
$(e_3, p_1, 1)$	1	1	1	1	1	1
$(e_3, p_1, 0)$	0	0	0	0	0	0
$(e_3, p_2, 1)$	0	0	1	1	1	1
$(e_3, p_2, 0)$	1	1	0	0	0	0

.

Example 3.2. *Let $G = S_3$, parameter $E = \{e_1, \cdots, e_3\}$, experts $X = \{p_1, p_2\}$, opinions $O = \{0, 1\}$ so $\mathcal{Z} = \{(e_i, p_j, o) \mid i = \{1, 2, 3\}, j = \{1, 2\}, o \in O\}$. Let $(\aleph, \mathcal{Z})_{S_3}$ be a SE-set defined by Table 1.*
If $\mathcal{L} = \{(e_1, p_1, 1), (e_2, p_2, 1), (e_3, p_1, 1), (e_1, p_2, 0), (e_3, p_2, 0)\}$, then $(\aleph, \mathcal{L})$ is a SE-group over S_3.

Definition 3.3. *In a FSE-set $(F_\mu, \mathcal{L})_G = \{(a, \{k_{\mu_{F(a)}(k)}, k \in F(a)\}), a \in \mathcal{L}\}$ is said to be fuzzy soft expert group (FSE-group), if for every $a \in \mathcal{L}$, $F_\mu(a) \leq_F G$.*

Table 2: FSE-set

F_μ	(1)	(1 2)	(1 3)	(2 3)	(1 2 3)	(1 3 2)
$(e_1, p_1, 1)$	0.8	0.5	0	0	0	0
$(e_1, p_1, 0)$	0	0	0.6	0.8	0.1	0.3
$(e_1, p_2, 1)$	0	1	0.3	0.1	0	0
$(e_1, p_2, 0)$	0.6	0	0	0	0.1	0.1
$(e_2, p_1, 1)$	0.9	0	0	0.7	0.4	0.1
$(e_2, p_1, 0)$	0	0.1	0.7	0	0	0
$(e_2, p_2, 1)$	1	0	0	0	1	1
$(e_2, p_2, 0)$	0	0.5	0.2	0.9	0	0
$(e_3, p_1, 1)$	0.8	0.5	0.5	0.5	0.6	0.6
$(e_3, p_1, 0)$	0	0	0	0	0	0
$(e_3, p_2, 1)$	0	0	0.3	0.5	0.1	0.6
$(e_3, p_2, 0)$	0.2	0	0	0.1	0	0

.

Example 3.4. *Let $G = S_3$, parameter $E = \{e_1, \cdots, e_3\}$, experts $X = \{p_1, p_2\}$, opinions $O = \{0, 1\}$ so $\mathcal{Z} = \{(e_i, p_j, o) \mid i = \{1, 2, 3\}, j = \{1, 2\}, o \in O\}$. FSE-set $(F_\mu, \mathcal{Z})_{S_3}$ defined by Table 2.*
Let $\mathcal{L} = \{(e_1, p_1, 1), (e_2, p_2, 1), (e_3, p_1, 1), (e_1, p_2, 0), (e_3, p_2, 0)\}$, then $(F_\mu, \mathcal{L})_{S_3}$ is a FSE-group.

Note that an IFSE-set $(F_\mu, F_\nu, \mathcal{L})$ as in Definition 2.7 can be written in the form $(F_\mu, F_\nu, \mathcal{L}) = \{(\theta, \{k_{\mu_{F(\theta)}(k)}, k \in F(\theta)\}, \{k_{\nu_{F(\theta)}(k)}, k \in F(\theta)\}), \theta \in \mathcal{L}\}$, with the condition $0 \leq \mu_{F(\theta)}(k) + \nu_{F(\theta)}(k) \leq 1$.

We generalize the idea of FSE-group to intuitionistic fuzzy setting in the Definition 3.5.

Definition 3.5. *In an intuitionistic FSE-set $(F_\mu, F_\nu, \mathcal{L})_G = \{(\theta, \{g_{\mu_{F(\theta)}(g)}, g \in F(\theta)\}, \{g_{\nu_{F(\theta)}(g)}, g \in F(\theta)\}), \theta \in \mathcal{L}\}$, if for every $\theta \in \mathcal{L}$, $F_\mu(\theta) \leq_F G$ and $(F_\nu(\theta))^c \leq_F G$, (i.e) $\mu_{F(\theta)}(g \cdot h) \geq min\{\mu_{F(\theta)}(g), \mu_{F(\theta)}(h)\}$, $\mu_{F(\theta)}(g^{-1}) = \mu_{F(\theta)}(g)$, $\nu_{F(\theta)}(g \cdot h) \leq max\{\nu_{F(\theta)}(g), \nu_{F(\theta)}(h)\}$ and $\nu_{F(\theta)}(g^{-1}) = \nu_{F(\theta)}(g)$, then $(F_\mu, F_\nu, \mathcal{L})_G$ is an intuitionistic fuzzy soft expert group (IFSE-group).*

Table 3: IFSE-set

F_μ, F_ν	(1)	(1 2)	(1 3)	(2 3)	(1 2 3)	(1 3 2)
$(e_1,p_1,1)$	0,0.9	0.8,0.1	0.3,0.5	0.1,0.8	0.1,0.6	0.2,0.7
$(e_1,p_2,1)$	0.8,0.2	0.5,0.5	0,1	0,1	0,1	0,1
$(e_2,p_1,1)$	0.9,0	0.2,0.6	0.1,0.8	0.7,0.2	0.5,0.5	0.1,0.7
$(e_2,p_2,1)$	0.2,0.8	0.3,0.5	0.3,0.6	0.5,0.3	0.1,0.5	0.6,0.3
$(e_3,p_1,1)$	0.9,0.1	0.5,0.5	0.5,0.5	0.5,0.5	0.7,0.3	0.7,0.3
$(e_3,p_2,1)$	1,0	0,1	0,1	0,1	1,0	1,0
$(e_1,p_1,0)$	0.9,0	0.1,0.8	0.6,0.3	0.8,0	0.6,0.3	0.6,0.1
$(e_1,p_2,0)$	0.1,0.7	0.5,0.4	0.7,0	0.1,0.7	0.4,0.4	0.7,0.1
$(e_2,p_1,0)$	0.6,0.4	0,1	0,1	0,1	0.1,0.9	0.1,0.9
$(e_2,p_2,0)$	0.7,0.1	0.5,0.3	0.5,0.3	0.3,0.5	0.1,0.6	0.2,0.6
$(e_3,p_1,0)$	0.1,0.8	0.5,0.4	0.4,0.5	0.6,0.2	0.3,0.6	0.5,0.2
$(e_3,p_2,0)$	0.2,0.8	0.1,0.9	0,1	0,1	0,1	0,1

.

Example 3.6. *Let $G = S_3$, parameter $E = \{e_1, \cdots, e_3\}$, experts $X = \{p_1, p_2\}$, opinions $O = \{0, 1\}$ so $\mathcal{Z} = \{(e_i, p_j, o)/i = \{1, 2, 3\}, j = \{1, 2\}, o \in O\}$. IFSE-set $(F_\mu, F_\nu, \mathcal{Z})_{S_3}$ is defined as in Table 3, the membership function F_μ of the element in S_3*

$F_\mu(e_1,p_1,1) = \{(1)_0, (1\ 2)_{0.8}, (1\ 3)_{0.3}, (2\ 3)_{0.1}, (1\ 2\ 3)_{0.1}, (1\ 3\ 2)_{0.2}\}$,
$F_\mu(e_1,p_2,1) = \{(1)_{0.8}, (1\ 2)_{0.5}, (1\ 3)_0, (2\ 3)_0, (1\ 2\ 3)_0, (1\ 3\ 2)_0\}$,
$F_\mu(e_2,p_1,1) = \{(1)_{0.9}, (1\ 2)_{0.2}, (1\ 3)_{0.1}, (2\ 3)_{0.7}, (1\ 2\ 3)_{0.5}, (1\ 3\ 2)_{0.1}\}$,
$F_\mu(e_2,p_2,1) = \{(1)_{0.2}, (1\ 2)_{0.3}, (1\ 3)_{0.3}, (2\ 3)_{0.5}, (1\ 2\ 3)_{0.1}, (1\ 3\ 2)_{0.6}\}$,
$F_\mu(e_3,p_1,1) = \{(1)_{0.9}, (1\ 2)_{0.5}, (1\ 3)_{0.5}, (2\ 3)_{0.5}, (1\ 2\ 3)_{0.7}, (1\ 3\ 2)_{0.7}\}$,
$F_\mu(e_3,p_2,1) = \{(1)_1, (1\ 2)_0, (1\ 3)_0, (2\ 3)_0, (1\ 2\ 3)_1, (1\ 3\ 2)_1\}$,
$F_\mu(e_1,p_1,0) = \{(1)_{0.9}, (1\ 2)_{0.1}, (1\ 3)_{0.6}, (2\ 3)_{0.8}, (1\ 2\ 3)_{0.6}, (1\ 3\ 2)_{0.6}\}$,
$F_\mu(e_1,p_2,0) = \{(1)_{0.1}, (1\ 2)_{0.5}, (1\ 3)_{0.7}, (2\ 3)_{0.1}, (1\ 2\ 3)_{0.4}, (1\ 3\ 2)_{0.7}\}$,
$F_\mu(e_2,p_1,0) = \{(1)_{0.6}, (1\ 2)_0, (1\ 3)_0, (2\ 3)_0, (1\ 2\ 3)_{0.1}, (1\ 3\ 2)_{0.1}\}$,
$F_\mu(e_2,p_2,0) = \{(1)_{0.7}, (1\ 2)_{0.5}, (1\ 3)_{0.5}, (2\ 3)_{0.3}, (1\ 2\ 3)_{0.1}, (1\ 3\ 2)_{0.2}\}$,
$F_\mu(e_3,p_1,0) = \{(1)_{0.1}, (1\ 2)_{0.5}, (1\ 3)_{0.4}, (2\ 3)_{0.6}, (1\ 2\ 3)_{0.3}, (1\ 3\ 2)_{0.5}\}$,
$F_\mu(e_3,p_2,0) = \{(1)_{0.2}, (1\ 2)_{0.1}, (1\ 3)_0, (2\ 3)_0, (1\ 2\ 3)_0, (1\ 3\ 2)_0\})\}$
and the non-membership functions F_ν of the element in S_3
$F_\nu(e_1,p_1,1) = \{(1)_{0.9}, (1\ 2)_{0.1}, (1\ 3)_{0.5}, (2\ 3)_{0.8}, (1\ 2\ 3)_{0.6}, (1\ 3\ 2)_{0.7}\}$,
$F_\nu(e_1,p_2,1) = \{(1)_{0.2}, (1\ 2)_{0.5}, (1\ 3)_1, (2\ 3)_1, (1\ 2\ 3)_1, (1\ 3\ 2)_1\}$,
$F_\nu(e_2,p_1,1) = \{(1)_0, (1\ 2)_{0.6}, (1\ 3)_{0.8}, (2\ 3)_{0.2}, (1\ 2\ 3)_{0.5}, (1\ 3\ 2)_{0.7}\}$,
$F_\nu(e_2,p_2,1) = \{(1)_{0.8}, (1\ 2)_{0.5}, (1\ 3)_{0.6}, (2\ 3)_{0.3}, (1\ 2\ 3)_{0.5}, (1\ 3\ 2)_{0.3}\}$,
$F_\nu(e_3,p_1,1) = \{(1)_{0.1}, (1\ 2)_{0.5}, (1\ 3)_{0.5}, (2\ 3)_{0.5}, (1\ 2\ 3)_{0.3}, (1\ 3\ 2)_{0.3}\}$,
$F_\nu(e_3,p_2,1) = \{(1)_0, (1\ 2)_1, (1\ 3)_1, (2\ 3)_1, (1\ 2\ 3)_0, (1\ 3\ 2)_0\}$,
$F_\nu(e_1,p_1,0) = \{(1)_0, (1\ 2)_{0.8}, (1\ 3)_{0.3}, (2\ 3)_0, (1\ 2\ 3)_{0.3}, (1\ 3\ 2)_{0.1}\}$,
$F_\nu(e_1,p_2,0) = \{(1)_{0.7}, (1\ 2)_{0.4}, (1\ 3)_0, (2\ 3)_{0.7}, (1\ 2\ 3)_{0.4}, (1\ 3\ 2)_{0.1}\}$,

$F_\nu(e_2,p_1,0)=\{(1)_{0.4},(1\ 2)_1,(1\ 3)_1,(2\ 3)_1,(1\ 2\ 3)_{0.9},(1\ 3\ 2)_{0.9}\}$,
$F_\nu(e_2,p_2,0)=\{(1)_{0.1},(1\ 2)_{0.3},(1\ 3)_{0.3},(2\ 3)_{0.5},(1\ 2\ 3)_{0.6},(1\ 3\ 2)_{0.6}\}$,
$F_\nu(e_3,p_1,0)=\{(1)_{0.8},(1\ 2)_{0.4},(1\ 3)_{0.5},(2\ 3)_{0.2},(1\ 2\ 3)_{0.6},(1\ 3\ 2)_{0.2}\}$,
$F_\nu(e_3,p_2,0)=\{(1)_{0.8},(1\ 2)_{0.9},(1\ 3)_1,(2\ 3)_1,(1\ 2\ 3)_1,(1\ 3\ 2)_1\})\}$
We can view the IFSE-set $(F_\mu,F_\nu,\mathcal{Z})_{S_3}$ *as*
$(F_\mu,F_\nu,\mathcal{Z})_{S_3}=\Big\{((e_1,p_1,1),\ \{(1)_{(0,0.9)},\ (1\ 2)_{(0.8,0.1)},\ (1\ 3)_{(0.3,0.5)},\ (2\ 3)_{(0.1,0.8)},$ $(1\ 2\ 3)_{(0.1,0.6)},\ (1\ 3\ 2)_{(0.2,0.7)}\}),\ ((e_1,p_2,1),\ \{(1)_{(0.8,0.2)},\ (1\ 2)_{(0.5,0.5)},\ (1\ 3)_{(0,1)},$ $(2\ 3)_{(0,1)},\ (1\ 2\ 3)_{(0,1)},\ (1\ 3\ 2)_{(0,1)}\}),\ ((e_2,p_1,1),\ \{(1)_{(0.9,0)},\ (1\ 2)_{(0.2,0.6)},\ (1\ 3)_{(0.1,0.8)},$ $(2\ 3)_{(0.7,0.2)},\ (1\ 2\ 3)_{(0.5,0.5)},\ (1\ 3\ 2)_{(0.1,0.7)}\}),\ ((e_2,p_2,1),\ \{(1)_{(0.2,0.8)},\ (1\ 2)_{(0.3,0.5)},$ $(1\ 3)_{(0.3,0.6)},\ (2\ 3)_{(0.5,0.3)},\ (1\ 2\ 3)_{(0.1,0.5)},\ (1\ 3\ 2)_{(0.6,0.3)}\}),\ ((e_3,p_1,1),\ \{(1)_{(0.9,0.1)},$ $(1\ 2)_{(0.5,0.5)},\ (1\ 3)_{(0.5,0.5)},\ (2\ 3)_{(0.5,0.5)},\ (1\ 2\ 3)_{(0.7,0.3)},\ (1\ 3\ 2)_{(0.7,0.3)}\}),\ ((e_3,p_2,1),$ $\{(1)_{(1,0)},\ (1\ 2)_{(0,1)},\ (1\ 3)_{(0,1)},\ (2\ 3)_{(0,1)},\ (1\ 2\ 3)_{(1,0)},\ (1\ 3\ 2)_{(1,0)}\}),\ ((e_1,p_1,0),$ $\{(1)_{(0.9,0)},\ (1\ 2)_{(0.1,0.8)},\ (1\ 3)_{(0.6,0.3)},\ (2\ 3)_{(0.8,0)},\ (1\ 2\ 3)_{(0.6,0.3)},\ (1\ 3\ 2)_{(0.6,0.1)}\}),$ $((e_1,p_2,0),\ \{(1)_{(0.1,0.7)},\ (1\ 2)_{(0.5,0.4)},\ (1\ 3)_{(0.7,0)},\ (2\ 3)_{(0.1,0.7)},\ (1\ 2\ 3)_{(0.4,0.4)},$ $(1\ 3\ 2)_{(0.7,0.1)}\}),\ ((e_2,p_1,0),\ \{(1)_{(0.6,0.4)},\ (1\ 2)_{(0,1)},\ (1\ 3)_{(0,1)},\ (2\ 3)_{(0,1)},\ (1\ 2\ 3)_{(0.1,0.9)},$ $(1\ 3\ 2)_{(0.1,0.9)}\}),\ ((e_2,p_2,0),\ \{(1)_{(0.7,0.1)},\ (1\ 2)_{(0.5,0.3)},\ (1\ 3)_{(0.5,0.3)},\ (2\ 3)_{(0.3,0.5)},$ $(1\ 2\ 3)_{(0.1,0.6)},\ (1\ 3\ 2)_{(0.2,0.6)}\}),\ ((e_3,p_1,0),\ \{(1)_{(0.1,0.8)},\ (1\ 2)_{(0.5,0.4)},\ (1\ 3)_{(0.4,0.5)},$ $(2\ 3)_{(0.6,0.2)},\ (1\ 2\ 3)_{(0.3,0.6)},\ (1\ 3\ 2)_{(0.5,0.2)}\}),\ ((e_3,p_2,0),\ \{(1)_{(0.2,0.8)},\ (1\ 2)_{(0.1,0.9)},$ $(1\ 3)_{(0,1)},\ (2\ 3)_{(0,1)},\ (1\ 2\ 3)_{(0,1)},\ (1\ 3\ 2)_{(0,1)}\})\Big\}$
Let $\mathcal{L}=\{(e_1,p_2,1),(e_3,p_1,1),(e_3,p_2,1),(e_2,p_1,0),(e_3,p_2,0)\}$, *then the membership function*
$(F_\mu(e_1,p_2,1)$, $(F_\mu(e_3,p_1,1)$, $(F_\mu(e_3,p_2,1)$, $(F_\mu(e_2,p_1,0)$, $(F_\mu(e_3,p_2,0)$ *and the non-membership functions*
$((F_\nu(e_1,p_2,1))^c=\{(1)_{0.8},(1\ 2)_{0.5},(1\ 3)_0,(2\ 3)_0,(1\ 2\ 3)_0,(1\ 3\ 2)_0\}$,
$((F_\nu(e_3,p_1,1))^c=\{(1)_{0.9},(1\ 2)_{0.5},(1\ 3)_{0.5},(2\ 3)_{0.5},(1\ 2\ 3)_{0.7},(1\ 3\ 2)_{0.7}\}$,
$((F_\nu(e_3,p_2,1))^c=\{(1)_1,(1\ 2)_0,(1\ 3)_0,(2\ 3)_0,(1\ 2\ 3)_1,(1\ 3\ 2)_1\}$,
$((F_\nu(e_2,p_1,0))^c=\{(1)_{0.6},(1\ 2)_0,(1\ 3)_0,(2\ 3)_0,(1\ 2\ 3)_{0.1},(1\ 3\ 2)_{0.1}\}$,
$((F_\nu(e_3,p_2,0))^c=\{(1)_{0.2},(1\ 2)_{0.1},(1\ 3)_0,(2\ 3)_0,(1\ 2\ 3)_0,(1\ 3\ 2)_0\})\}$ *are fuzzy subgroups in* S_3. *Hence* $(F_\mu,F_\nu,\mathcal{L})_{S_3}$ *is an IFSE-group.*

Definition 3.7. *The IFSE-set* $(F''_{\mu''},F''_{\nu''},\mathcal{L}'')_U$ *is said to be intersection of two IFSE-sets* $(F_\mu,F_\nu,\mathcal{L})_U$ *and* $(F'_{\mu'},F'_{\nu'},\mathcal{L}')_U$, *if* $\mathcal{L}''=\mathcal{L}\cap\mathcal{L}'$ *and every* $a\in\mathcal{L}''$, $k\in U$, $\mu''_{F''(a)}(k)=min\{\mu_{F(a)}(k),\mu'_{F'(a)}(k)\}$ *and* $\nu''_{F''(a)}(k)=max\{\nu_{F(a)}(k),\nu'_{F'(a)}(k)\}$ *and it is denoted by* $[(F_\mu,F_\nu,\mathcal{L})\tilde{\cap}(F'_{\mu'},F'_{\nu'},\mathcal{L}')]=(F''_{\mu''},F''_{\nu''},\mathcal{L}'')$.

Definition 3.8. *The IFSE-set* $(F''_{\mu''},F''_{\nu''},\mathcal{L}'')_U$ *is known as union of two IFSE-sets* $(F_\mu,F_\nu,\mathcal{L})_U$ *and* $(F'_{\mu'},F'_{\nu'},\mathcal{L}')_U$, *if* $\mathcal{L}''=\mathcal{L}\cup\mathcal{L}'$ *and* $\forall\ a\in\mathcal{L}''$, $k\in U$, $\mu''_{F''(a)}(k)=max\{\mu_{F(a)}(k),\mu'_{F'(a)}(k)\}$ *and* $\nu''_{F''(a)}(k)=min\{\nu_{F(a)}(k),\nu'_{F'(a)}(k)\}$. *It is denoted by* $(F''_{\mu''},F''_{\nu''},\mathcal{L}'')=(F_\mu,F_\nu,\mathcal{L})\tilde{\cup}(F'_{\mu'},F'_{\nu'},\mathcal{L}')$.

Definition 3.9. *If* $(F_\mu,F_\nu,\mathcal{L})_U$ *and* $(F'_{\mu'},F'_{\nu'},\mathcal{L}')_U$ *are two IFSE-sets, then* $(F_\mu,F_\nu,\mathcal{L})$ *AND* $(F'_{\mu'},F'_{\nu'},\mathcal{L}')$ *is defined by* $(F''_{\mu''},F''_{\nu''},\mathcal{L}\times\mathcal{L}')=(F_\mu,F_\nu,\mathcal{L})\wedge$

$(F'_{\mu'}, F'_{\nu'}, \mathcal{L}') = \{((\alpha,\beta), k_{\mu_{F(\alpha)}(k)\wedge\mu'_{F'(\beta)}(k)}, k_{\nu_{F(\alpha)}(k)\vee\nu'_{F'(\beta)}(k)})/k \in U, (\alpha,\beta) \in \mathcal{L}\times\mathcal{L}'\}$.

Definition 3.10. *If $(F_\mu, F_\nu, \mathcal{L})_U$ and $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_U$ are two IFSE-sets, then $(F_\mu, F_\nu, \mathcal{L})$ OR $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ is defined by $(F''_{\mu''}, F''_{\nu''}, \mathcal{L}\times\mathcal{L}') = (F_\mu, F_\nu, \mathcal{L}) \vee (F'_{\mu'}, F'_{\nu'}, \mathcal{L}') = \{((\alpha,\beta), k_{\mu_{F(\alpha)}(k)\vee\mu'_{F'(\beta)}(k)}, k_{\nu_{F(\alpha)}(k)\wedge\nu'_{F'(\beta)}(k)})/k \in U, (\alpha,\beta) \in \mathcal{L}\times\mathcal{L}'\}$.*

Theorem 3.11. *Let $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_G$ be two IFSE-groups. Then their intersection $[(F_\mu, F_\nu, \mathcal{L})\tilde{\cap}(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')]_G$ is an IFSE-group.*

Theorem 3.12. *Let $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_G$ be two IFSE-groups. If $\mathcal{L}\cap\mathcal{L}' = \emptyset$, then $(F_\mu, F_\nu, \mathcal{L})\tilde{\cup}(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ is an IFSE-group in G.*

Theorem 3.13. *Let $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_G$ be two IFSE-groups. Then $[(F_\mu, F_\nu, \mathcal{L}) \wedge (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')]_G$ is an IFSE-group.*

Definition 3.14. *Let μ be a fuzzy group of G, then the collection $\{x_{\mu_G(x)} \mid x = a^{-1}b^{-1}ab, a, b \in G\}$ is called the commutator fuzzy subgroup of G.*

Definition 3.15. *Let $(F_\mu, F_\nu, \mathcal{L})_G$ be an IFSE-group. If $\forall a \in \mathcal{L}$,*

(i) $\mu_{F(a)}(r) = \begin{cases} 1 & \text{if } r = e \\ 0 & \text{if } r \neq e \end{cases}$ *and $\nu_{F(a)}(r) = 0$, $\forall r \in G$, then $(F_\mu, F_\nu, \mathcal{L})_G$ is called an identity IFSE-group,*

(ii) $\mu_{F(a)}(r) = 1$ and $\nu_{F(a)}(r) = 0$, $\forall r \in G$, then $(F_\mu, F_\nu, \mathcal{L})_G$ is called an absolute IFSE-group,

(iii) $\mu_{F(a)}(r) = \begin{cases} 1 & \text{if } r \in Z(G) \\ 0 & \text{if } r \notin Z(G) \end{cases}$ *and $\nu_{F(a)}(r) = 0$, $\forall r \in G$, where $Z(G)$ is the center of the group G. Then $(F_\mu, F_\nu, \mathcal{L})_G$ is known as central IFSE-group,*

(iv) $F_\mu(r)$ is commutator fuzzy subgroup in G, then $(F_\mu, F_\nu, \mathcal{L})_G$ is known as commutator IFSE-group.

Theorem 3.16. *Let $f : G \to G'$ be a group homomorphism in an IFSE-group $(F_\mu, F_\nu, \mathcal{L})_G$. If $\forall r \in \mathcal{L}$ $F_\mu(r) \subseteq_F Kerf$, then $(fF_\mu, fF_\nu, \mathcal{L})_{G'}$ is an identity IFSE-group.*

Theorem 3.17. *Let f be a group homomorphism from G onto G'. If an IFSE-group $(F_\mu, F_\nu, \mathcal{L})_G$ is*

(i) an absolute IFSE-group, then $(fF_\mu, fF_\nu, \mathcal{L})_{G'}$ is an absolute IFSE-group,

(ii) central IFSE-group, then $(fF_\mu, fF_\nu, \mathcal{L})_{G'}$ is an central IFSE-group,

(iii) commutator IFSE-group, then $(fF_\mu, fF_\nu, \mathcal{L})_{G'}$ is commutator IFSE-group.

Corollary 3.18. *Let $(F_\mu, F_\nu, \mathcal{L})_G$ be an absolute (commutator or central) IFSE-group, then $(fF_\mu, fF_\nu, \mathcal{L})_{f(G)}$ is an absolute (commutator or central) IFSE-group.*

Definition 3.19. *Let $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_\omega, F'_\lambda, \mathcal{L}')_G$ be two IFSE-groups. Then $(F'_\omega, F'_\lambda, \mathcal{L}')$ is said to be an IFSE-subgroup of $(F_\mu, F_\nu, \mathcal{L})$, if $\mathcal{L}' \subseteq \mathcal{L}$ and $\omega_{F'(a)}(g) \leq_F \mu_{F(a)}(g)$ and $(\lambda_{F'(a)}(g))^c \leq_F (\nu_{F(a)}(g))^c$ $\forall a \in \mathcal{L}', g \in G$ and it is denoted by $(F'_\omega, F'_\lambda, \mathcal{L}') \tilde{<}_F (F_\mu, F_\nu, \mathcal{L})$.*

Example 3.20. *Let $G = S_3$, $\mathcal{Z}$ is defined as in example 3.6. IFSE-set $(F'_{\mu'}, F'_{\nu'}, \mathcal{Z})_{S_3}$ is defined by Table 4.*
Let $\mathcal{L}' = \{(e_1, p_2, 1), (e_3, p_1, 1), (e_2, p_1, 0), (e_3, p_2, 0)\}$, then $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_{S_3}$ is an IFSE-group. Also in Example 3.6 $(F_\mu, F_\nu, \mathcal{L})_{S_3}$ is an IFSE-group and $\mu'_{F'(e_1,p_2,1)}(g) \leq \mu_{F(e_1,p_2,1)}(g)$ $\nu'_{F'(e_1,p_2,1)}(g) \geq \nu_{F(e_1,p_2,1)}(g)$ $\mu'_{F'(e_3,p_1,1)}(g) \leq \mu_{F(e_3,p_1,1)}(g)$ $\nu'_{F'(e_3,p_1,1)}(g) \geq \nu_{F(e_3,p_1,1)}(g)$ $\mu'_{F'(e_2,p_1,0)}(g) \leq \mu_{F(e_2,p_1,0)}(g)$ $\nu'_{F'(e_2,p_1,0)}(g) \geq \nu_{F(e_2,p_1,0)}(g)$ $\mu'_{F'(e_3,p_2,0)}(g) \leq \mu_{F(e_3,p_2,0)}(g)$ $\nu'_{F'(e_3,p_2,0)}(g) \geq \nu_{F(e_3,p_2,0)}(g)$. Hence $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ is an IFSE-subgroup of $(F_\mu, F_\nu, \mathcal{L})$.

Table 4: IFSE-set

$F'_{\mu'}, F'_{\nu'}$	(1)	(1 2)	(1 3)	(2 3)	(1 2 3)	(1 3 2)
$(e_1, p_1, 1)$	0,1	0.7,0.3	0.2,0.7	0.1,0.9	0,1	0.1,0.8
$(e_1, p_2, 1)$	0.7,0.3	0.4,0.6	0,1	0,1	0,1	0,1
$(e_2, p_1, 1)$	0.8,0.2	0.1,0.8	0.1,0.9	0.4,0.5	0.3,0.6	0.1,0.8
$(e_2, p_2, 1)$	0.1,0.8	0.2,0.8	0,0.9	0.2,0.7	0,0.8	0.3,0.7
$(e_3, p_1, 1)$	0.8,0.2	0.4,0.6	0.4,0.6	0.4,0.6	0.5,0.5	0.5,0.5
$(e_3, p_2, 1)$	0.5,0.4	0,1	0,1	0,1	0.6,0.3	0.6,0.3
$(e_1, p_1, 0)$	0.7,0.2	0.05,0.9	0.4,0.5	0.6,0.1	0.3,0.5	0.5,0.5
$(e_1, p_2, 0)$	0.05,0.9	0.3,0.7	0.4,0.5	0,1	0.3,0.5	0.6,0.4
$(e_2, p_1, 0)$	0.5,0.5	0,1	0,1	0,1	0.05,0.9	0.05,0.9
$(e_2, p_2, 0)$	0.5,0.4	0.4,0.6	0.4,0.5	0.25,0.65	0.05,0.8	0.1,0.9
$(e_3, p_1, 0)$	0.05,0.8	0.4,0.45	0.3,0.8	0.4,0.5	0.2,0.8	0.3,0.8
$(e_3, p_2, 0)$	0.1,0.8	0.05,0.9	0,1	0,1	0,1	0,1

.

Theorem 3.21. *Let $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_G$ be two IFSE-groups,*

(i) if $\mathcal{L} = \mathcal{L}'$, $F_\mu(\gamma) \subseteq_F F'_{\mu'}(\gamma)$ and $F_\nu(\gamma) \subseteq_F F'_{\nu'}(\gamma)$, $\forall\ \gamma \in \mathcal{L}$, then $(F_\mu, F_\nu, \mathcal{L}) \tilde{<}_F (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$.

(ii) if $\mathcal{L} \subseteq \mathcal{L}'$, $F_\mu = F'_{\mu'}$ and $F_\nu = F'_{\nu'}$, then $(F_\mu, F_\nu, \mathcal{L})$ is an IFSE-subgroup of $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$.

Proof. (*i*) For every $\gamma \in \mathcal{L}$, we get $\mathcal{F}_{\mu}(\gamma) \leq_{\mathcal{F}} \mathcal{F}'_{\mu'}(\gamma)$ and $(\mathcal{F}'_{\nu'}(\gamma))^c \leq_{\mathcal{F}} (\mathcal{F}_{\nu}(\gamma))^c$. Hence $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L}) \tilde{<}_{\mathcal{F}} (\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$.
(*ii*) Let $\mathcal{L} \subseteq \mathcal{L}'$, so clearly for every $\gamma \in \mathcal{L}$, we get $\mathcal{F}_{\mu}(\gamma) \leq_{\mathcal{F}} \mathcal{F}'_{\mu'}(\gamma)$ and $(\mathcal{F}'_{\nu'}(\gamma))^c \leq_{\mathcal{F}} (\mathcal{F}_{\nu}(\gamma))^c$. □

As a consequence of Definitions 3.7, 3.8, 3.9 and 3.19, we introduce the Theorem 3.22.

Theorem 3.22. *Let $\{(\mathcal{F}_{\mu_i}, \mathcal{F}_{\nu i}, \mathcal{L}_i)\}_{i\in I}$ be a non-empty family of IFSE-subgroup in $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})$. Then*

1. *$\tilde{\bigcap}_{i\in I}(\mathcal{F}_{\mu_i}, \mathcal{F}_{\nu i}, \mathcal{L}_i)$ is an IFSE-subgroup of $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})$, if $\bigcap_{i\in I}\mathcal{L}_i \neq \emptyset$.*
2. *$\bigwedge_{i\in I}(\mathcal{F}_{\mu_i}, \mathcal{F}_{\nu i}, \mathcal{L}_i)$ is an IFSE-subgroup of $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})$,*
3. *$\tilde{\bigcup}_{i\in I}(\mathcal{F}_{\mu_i}, \mathcal{F}_{\nu i}, \mathcal{L}_i)$ is an IFSE-subgroup of $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})$, whenever $\mathcal{L}_i \cap \mathcal{L}_j = \emptyset$, $\forall i, j \in I$.*

Definition 3.23. *Two IFSE-subgroups $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ and $(\mathcal{F}''_{\mu''}, \mathcal{F}''_{\nu''}, \mathcal{L}'')$ of $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})_G$ are said to be conditionally IFSE-subgroup to each other, if either $\mathcal{F}'_{\mu'}(\beta) \leq_{\mathcal{F}} \mathcal{F}''_{\mu''}(\delta)$ and $(\mathcal{F}''_{\mu''}(\delta))^c \leq_{\mathcal{F}} (\mathcal{F}'_{\mu'}(\beta))^c$ (or) $\mathcal{F}''_{\mu''}(\delta) \leq_{\mathcal{F}} \mathcal{F}'_{\mu'}(\beta)$ and $(\mathcal{F}'_{\mu'}(\beta))^c \leq_{\mathcal{F}} (\mathcal{F}''_{\mu''}(\delta))^c$, whenever $\beta \in \mathcal{L}'$ and $\delta \in \mathcal{L}''$.*

Theorem 3.24. *Let $(\mathcal{F}''_{\psi}, \mathcal{F}''_{\tau}, \mathcal{L}'')$ and $(\mathcal{F}'_{\omega}, \mathcal{F}'_{\lambda}, \mathcal{L}')$ be two IFSE-subgroups of $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})_G$, then $(\mathcal{F}''_{\psi}, \mathcal{F}''_{\tau}, \mathcal{L}'')$ and $(\mathcal{F}'_{\omega}, \mathcal{F}'_{\lambda}, \mathcal{L}')$ are conditionally IFSE-subgroup to each other iff $(\mathcal{F}''_{\psi}, \mathcal{F}''_{\tau}, \mathcal{L}'') \vee (\mathcal{F}'_{\omega}, \mathcal{F}'_{\lambda}, \mathcal{L}')$ is an IFSE-subgroup of $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})$.*

Proof. Let $(\mathcal{F}''_{\mu''}, \mathcal{F}''_{\nu''}, \mathcal{L}'')$ and $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ be two IFSE-subgroups of $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})$. $(\mathcal{F}''_{\mu''}, \mathcal{F}''_{\nu''}, \mathcal{L}'')$ and $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ are conditionally IFSE-subgroup to each other $\Longleftrightarrow$ for every $\beta \in \mathcal{L}'$ and $\gamma \in \mathcal{L}''$ either $\mathcal{F}''_{\mu''}(\gamma)$ is a fuzzy subgroup of $\mathcal{F}'_{\mu'}(\beta)$ and $(\mathcal{F}'_{\mu'}(\beta))^c$ is a fuzzy subgroup of $(\mathcal{F}''_{\mu''}(\gamma))^c$ (or) $\mathcal{F}'_{\mu'}(\beta)$ is a fuzzy subgroup of $\mathcal{F}''_{\mu''}(\gamma)$ and $(\mathcal{F}''_{\mu''}(\gamma))^c$ is a fuzzy subgroup of $(\mathcal{F}'_{\mu'}(\beta))^c$
$\Longleftrightarrow \mathcal{F}'_{\mu'}(\beta) \cup \mathcal{F}''_{\mu''}(\gamma)$ and $(\mathcal{F}'_{\mu'}(\beta) \cap \mathcal{F}''_{\mu''}(\gamma))^c$ are fuzzy subgroups of G
$\Longleftrightarrow (\mathcal{F}''_{\mu''}, \mathcal{F}''_{\nu''}, \mathcal{L}'') \vee (\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ is an IFSE-subgroup of $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})$. □

Corollary 3.25. *Let $\{(\mathcal{F}_{\mu_i}, \mathcal{F}_{\nu i}, \mathcal{L}_i)\}_{i\in I}$ be a non-empty family of IFSE-subgroups in IFSE-group $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})_G$. Then $\bigvee_{i\in I}(\mathcal{F}_{\mu_i}, \mathcal{F}_{\nu i}, \mathcal{L}_i)$ is an IFSE-subgroup of $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})$ iff for every $i, j \in I$, $(\mathcal{F}_{\mu_i}, \mathcal{F}_{\nu i}, \mathcal{L}_i)$ and $(\mathcal{F}_{\mu_j}, \mathcal{F}_{\nu j}, \mathcal{L}_j)$ are conditionally IFSE-subgroup to each other.*

Theorem 3.26. *Let $(\mathcal{F}_{\mu}, \mathcal{F}_{\nu}, \mathcal{L})_G$, $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')_G$ and $(\mathcal{F}''_{\mu''}, \mathcal{F}''_{\nu''}, \mathcal{L}'')_{G'}$ be IFSE-groups. Then* (*i*) *if* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *be an IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$*, then* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *and* $(F_\mu, F_\nu, \mathcal{L})$ *are conditionally IFSE-subgroup to each other,*

(*ii*) *if* $(F_\mu, F_\nu, \mathcal{L})_G$ *be an identity IFSE-group, then* $(F_\mu, F_\nu, \mathcal{L})$ *and* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *are conditionally IFSE-subgroup to each other,*

(*iii*) *if* $(F_\mu, F_\nu, \mathcal{L})_G$ *be an absolute IFSE-group, then* $(F_\mu, F_\nu, \mathcal{L})$ *and* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *are conditionally IFSE-subgroup to each other,*

(*iv*) *if* $F_\mu(a) \subseteq_F Kerf$ $\forall a \in \mathcal{L}$*, where* f *being a group homomorphism from* G *to* G'*, then* $(fF_\mu, fF_\nu, \mathcal{L})$ *and* $(F''_{\mu''}, F''_{\nu''}, \mathcal{L}'')$ *are conditionally IFSE-subgroup to each other,*

(*v*) *if* $(F_\mu, F_\nu, \mathcal{L})_G$ *be an absolute IFSE-group, where* f *being a group homomorphism from* G *to* G'*, then* $(fF_\mu, fF_\nu, \mathcal{L})$ *and* $(F''_{\mu''}, F''_{\nu''}, \mathcal{L}'')$ *are conditionally IFSE-subgroup to each other.*

Proof. (*i*), (*ii*) and (*iii*) are trivially true from the definition of conditionally IFSE-subgroup.

(*iv*) Let $F_\mu(a) \subseteq_F Kerf$, then by Theorem 3.16 $(fF_\mu, fF_\nu, \mathcal{L})_{G'}$ is an identity IFSE-group, also by (*ii*) $(fF_\mu, fF_\nu, \mathcal{L})$ and $(F''_{\mu''}, F''_{\nu''}, \mathcal{L}'')$ are conditionally IFSE-subgroup to each other.

(*v*) Let $(F_\mu, F_\nu, \mathcal{L})_G$ be an absolute IFSE-group, then by Theorem 3.17 $(fF_\mu, fF_\nu, \mathcal{L})_{G'}$ is an absolute IFSE-group, also by (*iii*) $(fF_\mu, fF_\nu, \mathcal{L})$ and $(F''_{\mu''}, F''_{\nu''}, \mathcal{L}'')$ are conditionally IFSE-subgroup to each other. □

Now we define a product of IFSE-group as follows.

Definition 3.27. *Let* $(F_\mu, F_\nu, \mathcal{L})_G$ *and* $(F'_\omega, F'_\lambda, \mathcal{L}')_{G'}$ *be two IFSE-groups. Then the external direct product of IFSE-groups of* $(F_\mu, F_\nu, \mathcal{L})$ *and* $(F'_\omega, F'_\lambda, \mathcal{L}')_{G\times G'}$ *is an IFSE-group, defined as* $(F''_\psi, F''_\tau, \mathcal{L}\times\mathcal{L}') = (F_\mu, F_\nu, \mathcal{L}) \times (F'_\omega, F'_\lambda, \mathcal{L}')$*, where* $F''_\psi(a, a') = F_\mu(a) \times F'_\omega(a')$ *and* $F''_\tau(a, a') = F_\nu(a) \times F'_\lambda(a')$*,* $\forall (a, a') \in \mathcal{L}\times\mathcal{L}'$*.*

Definition 3.28. *The product of two IFSE-groups* $(F_\mu, F_\nu, \mathcal{L})_G$ *and* $(F'_\omega, F'_\lambda, \mathcal{L}')_G$ *is defined by* $(F''_\psi, F''_\tau, \mathcal{L}\times\mathcal{L}') = (F_\mu, F_\nu, \mathcal{L})(F'_\omega, F'_\lambda, \mathcal{L}')$*, where* $F''_\psi(a, a') = F_\mu(a)F'_\omega(a')$ *and* $F''_\tau(a, a') = F_\nu(a)F'_\lambda(a')$*,* $\forall (a, a') \in \mathcal{L}\times\mathcal{L}'$*.*

Theorem 3.29. *Let* $(F_{\mu_1}, F_{\nu_1}, \mathcal{L}_1)$ *and* $(F'_{\mu'_1}, F'_{\nu'_1}, \mathcal{L}'_1)$ *be two IFSE-subgroups of* $(F_\mu, F_\nu, \mathcal{L})_G$ *and* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_{G'}$ *respectively. Then the external direct product* $(F_{\mu_1}, F_{\nu_1}, \mathcal{L}_1) \times (F'_{\mu'_1}, F'_{\nu'_1}, \mathcal{L}'_1)$ *is an IFSE-subgroup of* $[(F_\mu, F_\nu, \mathcal{L}) \times (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')]_{G\times G'}$*.*

Proof. For every $(\alpha, \beta) \in \mathcal{L}_1 \times \mathcal{L}'_1$, $F_{\mu_1}(\alpha)$, $F'_{\mu'_1}(\beta)$, $(F_{\nu_1}(\alpha))^c$ and $(F'_{\nu'_1}(\beta))^c$ are fuzzy subgroups of $F_\mu(\alpha)$, $F'_{\mu'}(\beta)$, $(F_\mu(\alpha))^c$ and $(F'_{\mu'}(\beta))^c$ respectively, so $F_{\mu_1}(\alpha) \times F'_{\mu'_1}(\beta) \leq_F F_\mu(\alpha) \times F'_{\mu'}(\beta)$ and $(F_{\nu_1}(\alpha) \times F'_{\nu'_1}(\beta))^c = (F_{\nu_1}(\alpha))^c \times$

$(F'_{\nu'_1}(\beta))^c$ is a fuzzy subgroup of $(F_\mu(\alpha) \times F'_{\mu'}(\beta))^c$. Hence $(F_{\mu_1}, F_{\nu_1}, \mathcal{L}_1) \times (F'_{\mu'_1}, F'_{\nu'_1}, \mathcal{L}'_1)$ is an IFSE-subgroup of $[(F_\mu, F_\nu, \mathcal{L}) \times (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')]_{G\times G'}$. □

Theorem 3.30. *Let $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_\omega, F'_\lambda, \mathcal{L}')_{G'}$ be two IFSE-groups,*

(i) if both $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_\omega, F'_\lambda, \mathcal{L}')_{G'}$ are identity IFSE-groups over G and G' respectively, then $[(F_\mu, F_\nu, \mathcal{L}) \times (F'_\omega, F'_\lambda, \mathcal{L}')]_{G\times G'}$ is an identity IFSE-group,

(ii) if both $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_\omega, F'_\lambda, \mathcal{L}')_{G'}$ are absolute IFSE-groups, then $[(F_\mu, F_\nu, \mathcal{L}) \times (F'_\omega, F'_\lambda, \mathcal{L}')]_{G\times G'}$ is an absolute IFSE-group.

Proof. Proof is straight forward. □

Note that the product of two IFSE-groups in G is not an IFSE-group in G, in general.

Theorem 3.31. *Let $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_\omega, F'_\lambda, \mathcal{L}')_G$ be two IFSE-groups.*

(i) If any one of $(F_\mu, F_\nu, \mathcal{L})_G$ or $(F'_\omega, F'_\lambda, \mathcal{L}')_G$ is an identity IFSE-group, then $[(F_\mu, F_\nu, \mathcal{L})(F'_\omega, F'_\lambda, \mathcal{L}')]_G$ is an IFSE-group.

(ii) If any one of $(F_\mu, F_\nu, \mathcal{L})_G$ or $(F'_\omega, F'_\lambda, \mathcal{L}')_G$ is an absolute IFSE-group, then $[(F_\mu, F_\nu, \mathcal{L})(F'_\omega, F'_\lambda, \mathcal{L}')]_G$ is an absolute IFSE-group.

Proof. Proof is straight forward. □

Note that the product of two IFSE-subgroups need not be an IFSE-subgroup, in general.

Theorem 3.32. *Homomorphic image of an IFSE-group is an IFSE-group.*

Proof. Let $(F_\mu, F_\nu, \mathcal{L})_G$ be an IFSE-group, f being a group homomorphism from G to G', then for every $\gamma \in \mathcal{L}$, $f(F_\mu(\gamma)) \leq_F G'$, also $(f(F_\nu(\gamma)))^c = f((F_\nu(\gamma))^c) \leq_F G'$. Hence $(fF_\mu, fF_\nu, \mathcal{L})_{G'}$ is an IFSE-group. □

Corollary 3.33. *Let $f : G \to G'$ be a group homomorphism, if $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_G$ be two IFSE-groups with $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ be an IFSE-subgroup of $(F_\mu, F_\nu, \mathcal{L})$, then $(fF'_{\mu'}, fF'_{\nu'}, \mathcal{L}')$ is an IFSE-subgroup of $(fF_\mu, fF_\nu, \mathcal{L})$.*

Corollary 3.34. *Let $(F_\mu, F_\nu, \mathcal{L})_G$ and $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_G$ be two IFSE-groups with $(F_\mu, F_\nu, \mathcal{L})$ and $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ being conditionally IFSE-subgroup to each other, where f being a group homomorphism from G to G', then $(fF_\mu, fF_\nu, \mathcal{L})_G$ and $(fF'_{\mu'}, fF'_{\nu'}, \mathcal{L}')_{G'}$ are conditionally IFSE-subgroup to each other.*

$$
\begin{array}{ccc}
\mathcal{L} & \xrightarrow{\mathcal{F}_\mu} & I^G \\
{\scriptstyle\Delta}\big\downarrow & & {\scriptstyle\Gamma}\big\downarrow \\
\mathcal{L}' & \xrightarrow{\mathcal{F}'_{\mu'}} & I^{G'}
\end{array}
\qquad\qquad
\begin{array}{ccc}
\mathcal{L} & \xrightarrow{\mathcal{F}_\nu} & I^G \\
{\scriptstyle\Delta}\big\downarrow & & {\scriptstyle\Gamma}\big\downarrow \\
\mathcal{L}' & \xrightarrow{\mathcal{F}'_{\nu'}} & I^{G'}
\end{array}
$$

Figure 1: Intuitionistic Fuzzy soft expert function

Definition 3.35. *Let $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})_G$ and $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')_{G'}$ be two IFSE-groups, let Δ be a mapping from $\mathcal{L}$ to $\mathcal{L}'$ and Γ be a mapping from G to G', such that the Figure 1 commutes, that is $\Gamma\mathcal{F}_\mu = \mathcal{F}'_{\mu'}\Delta$ and $\Gamma\mathcal{F}_\nu = \mathcal{F}'_{\nu'}\Delta$. The pair $(\Gamma, \Delta) : (\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L}) \to (\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ is called IFSE-function.*

Note that if $G = \{h_1, h_2, \cdots\}$, then $\Gamma(G) = \{\Gamma(h_1), \Gamma(h_2), \cdots\}$.

Definition 3.36. *Let Γ be a homomorphism as in Definition 3.35. Then $(\Gamma, \Delta) : (\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L}) \to (\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ is called an IFSE-homomorphism, that is $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})$ is known as IFSE-homomorphic to $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$.*

Definition 3.37. *Let $(\Gamma, \Delta) : (\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L}) \to (\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ be an IFSE-homomorphism, then*

(i) the image of IFSE-homomorphism is defined by $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}'_I)$, where $\mathcal{L}'_I =$ image Δ,

(ii) the kernel of an IFSE-homomorphism is defined by $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L}_K)$, where $\mathcal{L}_K = \{r \in \mathcal{L} / \Gamma\mathcal{F}_\mu(r) = \{e'\}$ and $\Gamma\mathcal{F}^c_\nu(r) = \{e'\} \text{ in } G'\}$.

Theorem 3.38. *The image and kernel of an IFSE-homomorphism (Γ, Δ) : $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L}) \to (\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ are respectively IFSE-subgroups of $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ and $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})$.*

Definition 3.39. *Let $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})_G$ be an IFSE-group, then*

(i) $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})$ is called normal IFSE-group, if $\mathcal{F}_\mu(x) \trianglelefteq_F G$ and $(\mathcal{F}_\nu(x))^c \trianglelefteq_F G$, $\forall x \in \mathcal{L}$,

(ii) an IFSE-subgroup $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ of $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})$ is known as normal IFSE-subgroup of $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})$, if $\mathcal{F}'_{\mu'}(\beta) \trianglelefteq_F \mathcal{F}_\mu(\beta)$ and $(\mathcal{F}'_{\nu'}(\beta))^c \trianglelefteq_F (\mathcal{F}_\nu(\beta))^c$, $\forall \beta \in \mathcal{L}'$, and is denoted by $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}') \trianglelefteq_F (\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})$

(iii) an IFSE-subgroup $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ of $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})$ is known as identity IFSE-subgroup of $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})$, if $\mathcal{F}'_{\mu'}(y) = \{e\} = (\mathcal{F}'_{\nu'}(y))^c \ \forall y \in \mathcal{L}'$,

(iv) an IFSE-subgroup $(\mathcal{F}'_{\mu'}, \mathcal{F}'_{\nu'}, \mathcal{L}')$ of $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})$ is known as absolute IFSE-subgroup of $(\mathcal{F}_\mu, \mathcal{F}_\nu, \mathcal{L})$, if $\mathcal{F}'_{\mu'}(y) = G = (\mathcal{F}'_{\nu'}(y))^c \ \forall y \in \mathcal{L}'$,

(*v*) *an IFSE-subgroup* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *of* $(F_\mu, F_\nu, \mathcal{L})$ *is known as central IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$*, if* $F'_{\mu'}(y) = Z(G) = (F'_{\nu'}(y))^c$ $\forall y \in \mathcal{L}'$,

(*vi*) *an IFSE-subgroup* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *of* $(F_\mu, F_\nu, \mathcal{L})$ *is known as commutator IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$*, if* $F'_{\mu'}(y)$ *is a commutator fuzzy subgroup of* $F_\mu(y)$ *and* $(F'_{\nu'}(y))^c$ *is a commutator fuzzy subgroup of* $(F_\nu(y))^c$ $\forall y \in \mathcal{L}'$.

Theorem 3.40. *Let* $(F'_\omega, F'_\lambda, \mathcal{L}')$ *be an IFSE-subset of* $(F''_\psi, F''_\tau, \mathcal{L}'')$ *and* $(F''_\psi, F''_\tau, \mathcal{L}'') \tilde{<}_F (F_\mu, F_\nu, \mathcal{L})$*, if* $(F'_\omega, F'_\lambda, \mathcal{L}')$ *is a normal IFSE-subgroup in* $(F_\mu, F_\nu, \mathcal{L})$*, then* $(F'_\omega, F'_\lambda, \mathcal{L}')$ *is a normal IFSE-subgroup in* $(F''_\psi, F''_\tau, \mathcal{L}'')$.

Theorem 3.41. *If* $(F'_\omega, F'_\lambda, \mathcal{L}')$ *is an IFSE-subgroup of* $(F''_\psi, F''_\tau, \mathcal{L}'')$ *and* $(F''_\psi, F''_\tau, \mathcal{L}'')$ *is a normal IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$*, then* $(F'_\omega, F'_\lambda, \mathcal{L}') \wedge (F''_\psi, F''_\tau, \mathcal{L}'')$ *is a normal IFSE-subgroup of* $(F'_\omega, F'_\lambda, \mathcal{L}')$.

Proof. For every $(\delta, \gamma) \in \mathcal{L}' \times \mathcal{L}''$, by hypothesis we have $F'_\omega(\delta) \leq_F F''_\psi(\delta)$, $(F'_\lambda(\delta))^c \leq_F (F''_\tau(\delta))^c$, $F''_\psi(\gamma) \trianglelefteq_F F_\mu(\gamma)$ and $(F''_\tau(\gamma))^c$ is a fuzzy subgroup of $(F_\nu(\gamma))^c$, then $F'_\omega(\delta) \cap F''_\psi(\gamma) \trianglelefteq_F F'_\omega(\delta)$ and $(F'_\lambda(\delta) \cup F''_\tau(\gamma))^c = (F'_\lambda(\delta))^c \cap (F''_\tau(\gamma))^c$ is a fuzzy subgroup of $(F'_\lambda(\delta))^c$. Hence $(F'_\omega, F'_\lambda, \mathcal{L}') \wedge (F''_\psi, F''_\tau, \mathcal{L}'')$ is a normal IFSE-subgroup of $(F'_\omega, F'_\lambda, \mathcal{L}')$. □

Theorem 3.42. *Let* $(F_\mu, F_\nu, \mathcal{L})_G$ *be an IFSE-group, if* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is an identity (central or absolute or commutator) IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$*, then* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is a normal IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$.

Theorem 3.43. *Let* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_G$ *be an IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})_G$*, G being an abelian group, then*

(*i*) $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is a central IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$ *iff* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is an absolute IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$,

(*ii*) $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is a commutator IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$ *iff* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is an identity IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$,

(*iii*) $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is a normal IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$.

Theorem 3.44. *Let* $\{(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i)_G\}_{i\in I}$ *be a collection of normal IFSE-groups. Then*

(*i*) $\left[\tilde{\bigcap}_{i\in I}(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i)\right]_G$ *is a normal IFSE-group, if* $\bigcap_{i\in I} \mathcal{L}_i \neq \emptyset$,

(*ii*) $\left[\bigwedge_{i\in I}(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i)\right]_G$ *is a normal IFSE-group,*

(iii) $\left[\tilde{\bigcup\limits_{i\in I}}(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i)\right]_G$ *is a normal IFSE-group, whenever* $\mathcal{L}_i \cap \mathcal{L}_j = \emptyset$, $\forall i, j \in I$,

(iv) $\left[\bigvee\limits_{i\in I}(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i)\right]_G$ *is a normal IFSE-group iff for every* $i, j \in I$, $(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i)$ *and* $(F_{\mu_j}, F_{\nu j}, \mathcal{L}_j)$ *are conditionally IFSE-subgroup to each other.*

Theorem 3.45. *Let* $(F_{\mu_1}, F_{\nu_1}, \mathcal{L}_1)$ *and* $(F'_{\mu'_1}, F'_{\nu'_1}, \mathcal{L}'_1)$ *be two normal IFSE-subgroups of* $(F_\mu, F_\nu, \mathcal{L})$ *and* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *respectively. Then the external direct product* $(F_{\mu_1}, F_{\nu_1}, \mathcal{L}_1) \times (F'_{\mu'_1}, F'_{\nu'_1}, \mathcal{L}'_1)$ *is a normal IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L}) \times (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$.

Proof. From Theorem 3.29, proof is obvious. □

Theorem 3.46. *The product of two IFSE-subgroups of normal IFSE-group* $(F_\mu, F_\nu, \mathcal{L})_G$ *is an IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L} \times \mathcal{L})$ *if and only if either one of it is a normal IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$.

Proof. Let $(F'_\omega, F'_\lambda, \mathcal{L}')$ and $(F''_\psi, F''_\tau, \mathcal{L}'')$ be two IFSE-subgroups of a normal IFSE-group $(F_\mu, F_\nu, \mathcal{L})_G$, if its product $(F'_\omega, F'_\lambda, \mathcal{L}')(F''_\psi, F''_\tau, \mathcal{L}'')$ is an IFSE-subgroup of $(F_\mu, F_\nu, \mathcal{L} \times \mathcal{L})$, then for every $\beta \in \mathcal{L}'$, $\gamma \in \mathcal{L}''$, $F'_\omega(\beta)F''_\psi(\gamma)$ is a fuzzy subgroup of $F_\mu(\beta, \gamma)$ and $(F'_\lambda(\beta)F''_\tau(\gamma))^c$ is a fuzzy subgroup of $(F_\nu(\beta, \gamma))^c$. Also if both $(F'_\omega, F'_\lambda, \mathcal{L}')$ and $(F''_\psi, F''_\tau, \mathcal{L}'')$ are not normal IFSE-subgroups of $(F_\mu, F_\nu, \mathcal{L})$, then there is some $\beta \in \mathcal{L}'$ and $\gamma \in \mathcal{L}''$ with either $F'_\omega(\beta)F''_\psi(\gamma)$ is not a fuzzy subgroup of $F_\mu(\beta, \gamma)$ or $(F'_\lambda(\beta)F''_\tau(\gamma))^c = (F'_\lambda(\beta))^c(F''_\tau(\gamma))^c$ is not a fuzzy subgroup of $(F_\nu(\beta, \gamma))^c$. Hence either one of $(F'_\omega, F'_\lambda, \mathcal{L}')$ and $(F''_\psi, F''_\tau, \mathcal{L}'')$ is a normal IFSE-subgroup of $(F_\mu, F_\nu, \mathcal{L})$.

Conversely assume $(F'_\omega, F'_\lambda, \mathcal{L}')$ is a normal IFSE-subgroup of $(F_\mu, F_\nu, \mathcal{L})$, so $F'_\omega(\beta) \trianglelefteq_F F_\mu(\beta)$ and $(F'_\lambda(\beta))^c \trianglelefteq_F (F_\nu(\beta))^c$, $\forall \beta \in \mathcal{L}'$. Now for every $\gamma \in \mathcal{L}''$, $F''_\psi(\gamma) \leq_F F_\mu(\gamma)$ and $(F''_\tau(\gamma))^c \leq_F (F_\nu(\gamma))^c$. So $F'_\omega(\beta)F''_\psi(\gamma) \leq_F F_\mu(\beta)F_\mu(\gamma)$ and $(F'_\lambda(\beta)F''_\tau(\gamma))^c$ is a fuzzy subgroup of $(F_\nu(\beta)F_\nu(\gamma))^c$. □

Corollary 3.47. *The product of two normal IFSE-groups in* G *is a normal IFSE-group in* G.

The Theorem 3.48 is a consequence of Definition 3.7, 3.8, 3.9 and 3.39.

Theorem 3.48. *Let* $\{(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i)\}_{i\in I}$ *be a non-empty family of normal IFSE-subgroups in* $(F_\mu, F_\nu, \mathcal{L})_G$. *Then*

1. $\tilde{\bigcap\limits_{i\in I}}(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i) \trianglelefteq_F (F_\mu, F_\nu, \mathcal{L})$, *if* $\bigcap\limits_{i\in I}\mathcal{L}_i \neq \emptyset$,

2. $\bigwedge_{i\in I}(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i) \trianglelefteq_F (F_\mu, F_\nu, \mathcal{L})$,

3. $\tilde{\bigcup}_{i\in I}(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i) \trianglelefteq_F (F_\mu, F_\nu, \mathcal{L})$, *whenever* $\mathcal{L}_i \cap \mathcal{L}_j = \emptyset$, $\forall i, j \in I$,

4. $\bigvee_{i\in I}(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i)$ *is a normal IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$ *iff for every* $i, j \in I$, $(F_{\mu_i}, F_{\nu i}, \mathcal{L}_i)$ *and* $(F_{\mu_j}, F_{\nu j}, \mathcal{L}_j)$ *are conditionally IFSE-subgroup to each other.*

Theorem 3.49. *The kernel of IFSE-homomorphism* $(\Gamma, \Delta) : (F_\mu, F_\nu, \mathcal{L}) \to (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is a normal IFSE-subgroup in* $(F_\mu, F_\nu, \mathcal{L})$.

Definition 3.50. *An agree-IFSE-group* $(F_\mu, F_\nu, \mathcal{L})_1$ *and disagree-IFSE-group* $(F_\mu, F_\nu, \mathcal{L})_0$ *over* U *are IFSE-subgroups of* $(F_\mu, F_\nu, \mathcal{L})_U$ *are respectively defined as* $(F_\mu, F_\nu, \mathcal{L})_1 = \{(a, F_\mu(a), F_\nu(a)), a \in \mathcal{L} \cap (E \times X \times \{1\})\}$ *and* $(F_\mu, F_\nu, \mathcal{L})_0 = \{(a, F_\mu(a), F_\nu(a)), a \in \mathcal{L} \cap (E \times X \times \{0\})\}$.

Note that for an IFSE-homomorphism $(\Gamma, \Delta) : (F_\mu, F_\nu, \mathcal{L}) \to (F'_\omega, F'_\lambda, \mathcal{L}')$**, the IFSE-group** $(F_\mu, F_\nu, \mathcal{L})$**, kernel of** (Γ, Δ) **and image of** (Γ, Δ) **are respectively** $(F_\mu, F_\nu, \mathcal{L}) = \{(\theta, F_\mu(\theta), F_\nu(\theta)), \theta \in \mathcal{L}\} = \{(\theta, \{k_{\mu_{F(\theta)}(k)}, k \in F(\theta)\}, \{k_{\nu_{F(\theta)}(k)}, k \in F(\theta)\}), \theta \in \mathcal{L}\}$**,** $(F_\mu, F_\nu, \mathcal{L}_K) = \{(\theta, F_\mu(\theta), F_\nu(\theta)), \theta \in \mathcal{L}_K\} = \{(\theta, \{k_{\mu_{F(\theta)}(k)}, k \in F(\theta)\}, \{k_{\nu_{F(\theta)}(k)}, k \in F(\theta)\}), \theta \in \mathcal{L}_K\}$ **and** $(F'_\omega, F'_\lambda, \mathcal{L}'_I) = \{(\gamma, F'_\omega(\gamma), F'_\lambda(\gamma)), \gamma \in \mathcal{L}'_I\} = \{(\gamma, \{s_{\mu_{F'(\gamma)}(s)}, s \in F'(\gamma)\}, \{s_{\nu_{F(\gamma)}(s)}, s \in F'(\gamma)\}), \gamma \in \mathcal{L}'_I\}$**.**

Definition 3.51. *Let* $(F_\mu, F_\nu, \mathcal{L})_U$ *be an IFSE-group with* $F_\mu(x_1) = F_\mu(x_2)$ *or* $F_\nu(x_1) = F_\nu(x_2)$ *for some* $x_1, x_2 \in \mathcal{L}$, $x_1 \neq x_2$. *We define a restriction of IFSE-group* F_μ, F_ν *to* $\mathcal{L}^R$, $(F_\mu, F_\nu, \mathcal{L}^R) = \{(x, F_\mu(x), F_\nu(x)) \in (F_\mu, F_\nu, \mathcal{L}), x \in \mathcal{L}^R\}$, *using distinct parameter in* $\mathcal{L}$ *that has different images, where* $\mathcal{L}^R$ *is maximal fuzzy subset of* $\mathcal{L} \ni$ *for every* $\alpha, \beta \in \mathcal{L}^R$, $F_\mu(\alpha) \neq F_\mu(\beta)$ *and* $F_\nu(\alpha) \neq F_\nu(\beta)$. *Clearly* $(F_\mu, F_\nu, \mathcal{L}^R)$ *is an IFSE-subgroup of* $(F_\mu, F_\nu, \mathcal{L})$.

Definition 3.52. *If* Γ *is a monomorphism and* Δ *is one to one in Definition 3.35, then* $(\Gamma, \Delta) : (F_\mu, F_\nu, \mathcal{L}) \to (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is an IFSE-monomorphism. If* Γ *is an epimorphism and* Δ *is onto in Definition 3.35, then* $(\Gamma, \Delta) : (F_\mu, F_\nu, \mathcal{L}) \to (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is an IFSE-epimorphism.*

Theorem 3.53. *Let* $(F_\mu, F_\nu, \mathcal{L})_G$ *and* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_{G'}$ *be IFSE-groups and* $(\Gamma, \Delta) : (F_\mu, F_\nu, \mathcal{L}) \to (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *be an IFSE-homomorphism.*

(i) If $(\Gamma, \Delta) : (F_\mu, F_\nu, \mathcal{L}) \to (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is an IFSE-monomorphism, then* $(F'_{\mu'}, F'_{\nu'}, \Delta(\mathcal{L}_K))_{G'}$ *is an identity IFSE-group,*

(ii) If $(\Gamma, \Delta) : (F_\mu, F_\nu, \mathcal{L}) \to (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is an IFSE-monomorphism, then* $(F_\mu, F_\nu, \mathcal{L}_K)_G$ *is an identity IFSE-group,*

(iii) *If* $(\Gamma, \Delta) : (F_\mu, F_\nu, \mathcal{L}) \to (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is an IFSE-monomorphism and* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_{G'}$ *is an identity IFSE-group, then* $(F_\mu, F_\nu, \mathcal{L})_G$ *is an identity IFSE-group,*

(iv) *If* $(F_\mu, F_\nu, \mathcal{L})_G$ *is an identity IFSE-group, then* $(F'_{\mu'}, F'_{\nu'}, \Delta(a))_{G'}$ *is an identity IFSE-group,*

(v) *If* $(\Gamma, \Delta) : (F_\mu, F_\nu, \mathcal{L}) \to (F'_{\mu'}, F'_{\nu'}, \mathcal{L}')$ *is an IFSE-epimorphism and* $(F_\mu, F_\nu, \mathcal{L})_G$ *is an absolute IFSE-group, then* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_{G'}$ *is an absolute IFSE-group,*

(vi) *If* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_{G'}$ *is an absolute IFSE-group, then* $(F_\mu, F_\nu, \mathcal{L})_G$ *is an absolute IFSE-group,*

(vii) *If* $(F'_{\mu'}, F'_{\nu'}, \mathcal{L}')_{G'}$ *is a central (commutator) IFSE-group, then* $(F_\mu, F_\nu, \mathcal{L})_G$ *is a central (commutator) IFSE-group,*

(viii) *If* $(F_\mu, F_\nu, \mathcal{L})_G$ *is a central (commutator) IFSE-group, then* $(F'_{\mu'}, F'_{\nu'}, \Delta(\mathcal{L}))_{\Gamma(G)}$ *is a central (commutator) IFSE-group.*

4 Conclusion

The concept of IFSE-groups was summarised in this paper. Theoretical aspects of IFSE-sets were used to present this new algebraic structure. IFSE-subgroup, identity (absolute, central, commutator, normal) IFSE-subgroup of IFSE-group, and IFSE-homomorphism have been the focused on this paper. In the future, we intend to expand on our work on IFSE-set features in various algebraic structures.

5 Open Problem

This paper describes about intuitionistic fuzzy soft expert group with interesting results on it. The following open problems may be solved that arose from this paper.

(i) Is it possible to construct soft-rough expert groups.

(ii) Is it possible to construct intuitionistic fuzzy soft expert modules over a fuzzy soft expert ring.

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