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A study of Cellular Neural Networks with New Vertex-Edge Topological indices

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Abstract

In the field of image processing, non-linear processing, geometric maps, high-speed computations cellular neural network, or Cellular Non-linear Network (CNN) has various parallel processing applications. It is similar to neural networks, but the difference is communication is allowed between the neighboring cells only. An array of Cells are interconnected locally and can be arranged in a different configuration. Each cell consists of an input, a state, an output, and graphically, cells are represented by a vertex and their connections with the neighboring cell are represented by edges. Nowadays, topological descriptors (single values that characterize the graph) are used in chemical graph theory for the study of graph structure and its biological properties. In this article, we study newly defined ve-degree version of the Sombor index,Nirmala index, and Misbalance prodeg index and its topological properties of cellular neural networks. The results are helpful in the applications of cellular neural networks.

Keywords: *Cellular Non-linear network, vertex-edge degree; topological indices*

1 Introduction

Graph theory played a significant role in molecular chemistry, robotics, physics, networks computer science, statistics, biological activities, and data science.

A topological index is a unique number that is mathematically derived from the graph structure. In theoretical chemistry, many such topological indices have been considered, and have more applications in a quantitative structure-property relationship ($QSPR$) and quantitative structure-activity relationship ($QSAR$). The ($QSPR$)/($QSAR$) studies have an important role in material sciences [1, 2, 3]. The vertex-edge topological indices is a new idea and recently gaining more interest in applied sciences [4, 5, 6, 7, 8, 9, 10, 11, 12].

Let $G = (V, E)$ be a simple connected graph. The number of edges that are incident with the vertex u is known as the degree of the vertex u and is denoted by, $d(u)$. In [5], the set $N(u) = \{u \in V(G) \ : uw \in E(G)\}$ and $N[u] = N(u) \cup \{u\}$ are called as open and closed neighbourhood of the vertex u. The number of different edges that are incident to any vertex from $N[u]$, denoted by $d_{ve}(u)$ and called as ve-degree. Recently, the ve-degree Sombor index (SO_{ve}) [13], the ve-degree Nirmala index (N_{ve}) [14], and the ve-degree Misbalance prodeg index (MPI_{ve}) [14] as follows:

$$SO_{ve} = \sum_{uw \in E} \sqrt{d_{ve}(u)^2 + d_{ve}(w)^2}, \ N_{ve} = \sum_{uw \in E} \sqrt{d_{ve}(u) + d_{ve}(w)},$$

$$MPI_{ve} = \sum_{uw \in E} \left(\sqrt{d_{ve}(u)} + \sqrt{d_{ve}(w)} \right)$$

respectively. In the next section, we will discuses the motivation of this study, application, and importance of cellular neural network (CNN).

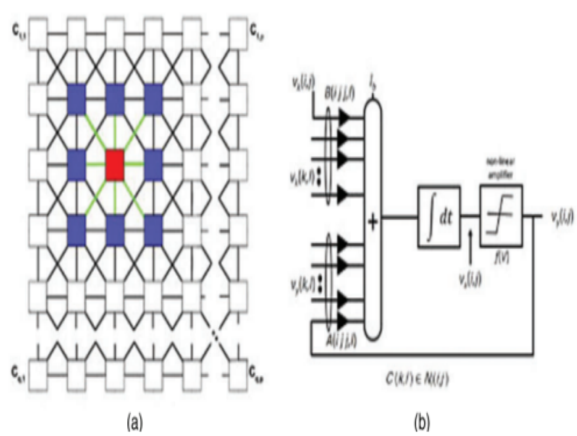

Figure 1: Graph of cellular neural network. (a) 2D graph of CNN, (b) red cell internal structure of (a).

2 Applications and Importance of CNN

In 1988, Chua and Yang proposed a cellular neural network (CNN) which is more general than Hopfield neural network [15, 16]. The main characteristic of CNN is parallel processing which means, the states of all cells can be evaluated at the same time. This is the reason behind the very fast operation speed. The cells are connected locally, so the CNN is suited for realizing with VLSI. The structure of the cellular neural network is regular, parallel array, local connection, and also suitable for the integrated circuit. It also helps in some special calculations that have the characteristic of local regular connection. The diagram (a) of Fig 1 represents the $2D$ graph of CNN in which a neighborhood is representing red and blue colors and diagram (b) of Fig 1 represents the internal structure of the red cell of diagram (a), this cell can interact with all the blue cells in its neighborhood. The CNN can be used in machine learning algorithm [17]. The CNN has great potential to study partial differential equations [18, 19]. It also has wide applications in analyzing $3D$ surfaces and facial images [20]. The CNN also has importance in artificial intelligence [21].

In literature [22, 23, 24, 25, 26, 27], it is found that vertex-edge based (ve-degree) topological indices are considered and studied in neural networks and Cellular neural networks. Topological indices have been studied and given prominence to their importance, but newly defined topological indices have not been considered for this study. Seeing the importance of Cellular neural networks, motivated us to, define new vertex-edge topological descriptors. The results are generalized and can be used for CNN of any structure and size. This will enhance the applications of CNN in image processing, parallel processing, image processing, non-linear processing, geometric maps, high-speed computations, solving partial differential equations, analyzing 3D surfaces, sensory-motor organs, and modeling biological vision [22].

3 The graph of Cellular neural network

The Cellular neural network can be arranged in a linearly form. If CNN has p rows and q columns, then it contains pq vertices and $4pq - 3p - 3q + 2$ edges. The CNN graph for $p = 7$ and $q = 7$ along with sum of degree of neighborhood vertices is shown in Fig 2. The vertex and edge partition of CNN based on ve-degree are shown in Table 1 and 2.

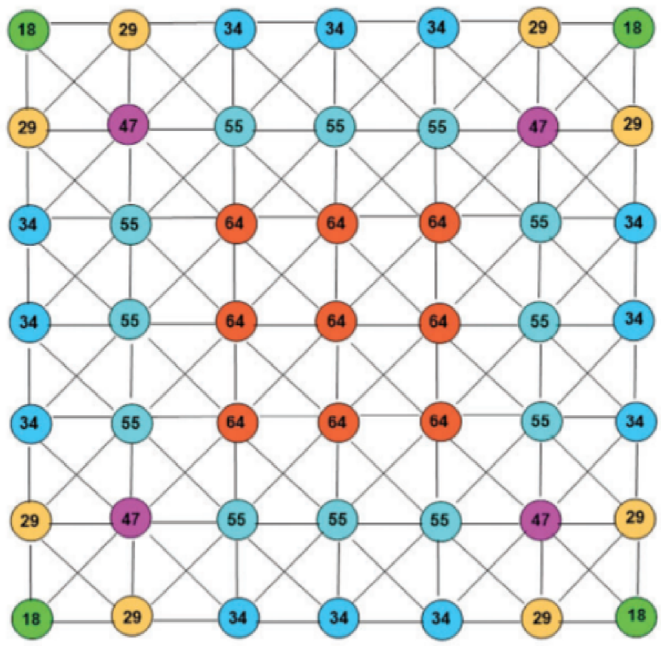

Figure 2: The CNN graph for p = 7 and q = 7 along with sum of degrees of neighborhood vertices.

4 Main results

In the following sections, we discuss the values of $SO_{ve},\ N_{ve}, \text{and} MPI_{ve}$ for CNN.

4.1 The ve-degree Sombor index (SO_{ve})

We partitioned the vertices of cellular neural network (CNN) according to its ve-degree that is shown in Table 2. Using this, we compute the ve-degree

Table 1: Vertex partition of CNN, based on ve-degree

Number of vertices	$d_{ve}(u)$	$d(u)$
4	18	3
8	29	5
2 (p + q)-16	34	5
4	47	8
2(p + q)-16	55	8
(p - 4)(q-4)	64	8

Table 2: Edge partition of CNN, based on ve-degree

Number of edges	$(d_{ve}(u), d_{ve}(w))$	$(d(u), d(w))$
8	(18, 29)	(3, 5)
4	(18, 47)	(3, 8)
4	(29, 29)	(5, 5)
8	(29, 34)	(5, 5)
$2(p-5)+2(q-5)$	(34, 34)	(5, 5)
8	(29, 47)	(5, 8)
8	(29, 55)	(5, 8)
8	(34, 47)	(5, 8)
$2(3p-14)+2(3q-14)$	(34, 55)	(5, 8)
8	(47, 55)	(8, 8)
4	(47, 64)	(8, 8)
$2(p+q-8)$	(55, 55)	(8, 8)
$6(p-6)+6(q-6)+16$	(55, 64)	(8, 8)
$2(p-4)(q-4)$	(64, 64)	(8, 8)

Sombor index (SO_{ve}).

$$
\begin{aligned}
SO_{ve} = &\sum_{uw\in E} \sqrt{d_{ve}(u)^2 + d_{ve}(w)^2}\\
=&8(\sqrt{18^2+29^2}) + 4(\sqrt{18^2+47^2}) + 4(\sqrt{29^2+29^2}) + 8(\sqrt{29^2+34^2})\\
&+ (2p+2q-20)(\sqrt{34^2+34^2}) + 8(\sqrt{29^2+47^2}) + 8(\sqrt{29^2+55^2})\\
&+ 8(\sqrt{34^2+47^2}) + (6p+6q-56)(\sqrt{34^2+55^2}) + 8(\sqrt{47^2+55^2})\\
&+ 4(\sqrt{47^2+64^2}) + (2p+2q-16)(\sqrt{55^2+55^2})\\
&+ (6p+6q-56)(\sqrt{55^2+64^2}) + (2pq-8q-8q+32)(\sqrt{64^2+64^2})\\
=&128\sqrt{2}pq + (p+q)[6\sqrt{4181} + 6\sqrt{7121} - 334\sqrt{2}] + 8[\sqrt{1165} + \sqrt{1997}\\
&+ \sqrt{3050} + \sqrt{3866} + \sqrt{3365} + \sqrt{5234}] + 4[\sqrt{2533} + \sqrt{6305}]\\
&- 56[\sqrt{4181} + \sqrt{7121}] + 604\sqrt{2}.
\end{aligned}
$$

4.2 The ve-degree Nirmala index (N_{ve})

Using Table 2, we compute the ve-degree Nirmala index (N_{ve}).

$$
\begin{aligned}
N_{ve} = \sum_{uw\in E} & \sqrt{d_{ve}(u)+d_{ve}(w)} \\
=& 8(\sqrt{18+29})+4(\sqrt{18+47})+4(\sqrt{29+29})+8(\sqrt{29+34}) \\
&+(2p+2q-20)(\sqrt{34+34})+8(\sqrt{29+47})+8(\sqrt{29+55}) \\
&+8(\sqrt{34+47})+(6p+6q-56)(\sqrt{34+55})+8(\sqrt{47+55}) \\
&+4(\sqrt{47+64})+(2p+2q-16)(\sqrt{55+55})+(6p+6q-56)(\sqrt{55+64}) \\
&+(2pq-8q-8q+32)(\sqrt{64+64}) \\
=& (p+q)[2\sqrt{68}+6\sqrt{89}+2\sqrt{110}+6\sqrt{119}-8\sqrt{128}]+8[\sqrt{47}+\sqrt{63} \\
&+\sqrt{76}+\sqrt{84}+\sqrt{102}+9]+4[\sqrt{65}+\sqrt{58}]-20\sqrt{68}-56\sqrt{89} \\
&-16\sqrt{110}-56\sqrt{119}+32\sqrt{128}+2\sqrt{128}pq.
\end{aligned}
$$

4.3 The ve-degree Misbalance prodeg index (MPI_{ve})

By using Table 2, we compute the ve-degree Misbalance prodeg index (MPI_{ve}).

$$
\begin{aligned}
MPI_{ve} = \sum_{uw\in E} & \left(\sqrt{d_{ve}(u)}+\sqrt{d_{ve}(w)}\right) \\
=& 8(\sqrt{18}+\sqrt{29})+4(\sqrt{18}+\sqrt{47})+4(\sqrt{29}+\sqrt{29})+8(\sqrt{29}+\sqrt{34}) \\
&+(2p+2q-20)(\sqrt{34}+\sqrt{34})+8(\sqrt{29}+\sqrt{47})+8(\sqrt{29}+\sqrt{55}) \\
&+8(\sqrt{34}+\sqrt{47})+(6p+6q-56)(\sqrt{34}+\sqrt{55})+8(\sqrt{47}+\sqrt{55}) \\
&+4(\sqrt{47}+\sqrt{64})+(2p+2q-16)(\sqrt{55}+\sqrt{55}) \\
&+(6p+6q-56)(\sqrt{64}+\sqrt{55})+(2pq-8q-8q+32)(\sqrt{64}+\sqrt{64}) \\
=& 32pq+(p+q)[10\sqrt{34}+16\sqrt{55}-80]+36\sqrt{2}+40\sqrt{29}+32\sqrt{47}+64 \\
&-128\sqrt{55}-80\sqrt{34}.
\end{aligned}
$$

5 Conclusion

The new vertex-edge topological indices namely, SO_{ve}, N_{ve}, and MPI_{ve} are calculated for the Cellular Neural Networks. Further, we compare the numerical values of the SO_{ve}, N_{ve}, and MPI_{ve} for the CNN and are shown in the Table 3. The graphical representation of those values can be seen in Fig 3. The found results can be used for any size of the CNN and those results can enhance the CNN's applications in image processing, parallel processing, non-linear processing, geometric maps representations, high-speed computations, solving partial differential equations, analyzing $3D$ surfaces, sensory-motor organs, and modeling biological vision.

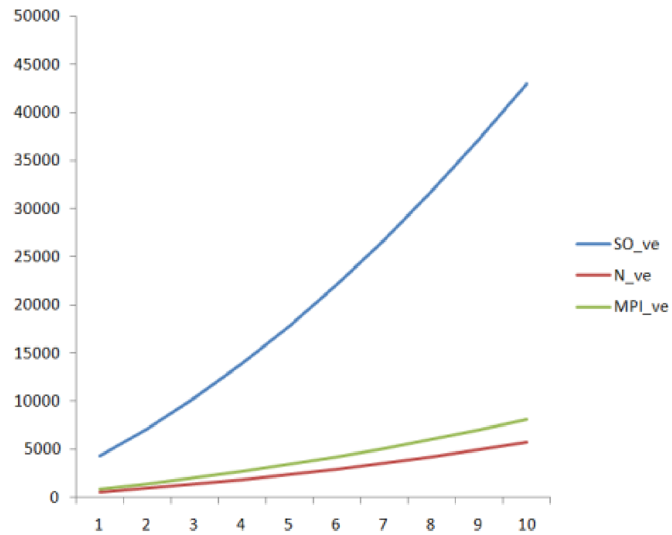

Figure 3: Graphical comparison of the SO_{ve}, N_{ve}, and MPI_{ve}.

Table 3: Numerical comparison of SO_{ve}, N_{ve}, and MPI_{ve}

$[p, q]$	SO_{ve}	N_{ve}	MPI_{ve}
$[5, 5]$	4383.9446	622.8655	903.6366
$[6, 6]$	7219.0227	1009.7973	1449.574
$[7, 7]$	10416.1395	1441.9839	2059.5113
$[8, 8]$	13975.2948	1919.4253	2733.4487
$[9, 9]$	17896.4889	2442.1216	3471.3861
$[10, 10]$	22179.7217	3010.0727	4273.3235
$[11, 11]$	26824.9932	3623.2787	5139.2609
$[12, 12]$	31832.3033	4281.7394	6069.1983
$[13, 13]$	37201.6521	4985.455	7063.1357
$[14, 14]$	42933.0395	5734.4251	8121.0731

6 Open Problem

Further,one can extend this work with some more topological indices (like,reverse topological index) and compare the results. Those results can be found helpful in image processing, parallel processing and further more.

References

[1] A. Ahmad, On the degree based topological indices of benzene ring embedded in P-type-surface in 2D network, *Hacettepe Journal of Mathematics and Statistics*, vol. 47, no. 1, (2018), pp. 9-18.

[2] M. Karelson, Molecular Descriptors in QSAR/QSPR. *New York: Wiley*, 2000.

[3] R. Todeschini and V. Consonni, *Handbook of Molecular Descriptors. Weinheim: Wiley*, 2000.

[4] T. Došlic, B. Furtula, A. Graovac, I. Gutman, S. Moradid, On vertex-degree-based molecular structure descriptors, *MATCH Communications in Mathematical and in Computer Chemistry*, vol. 66, (2011),pp.613-626.

[5] M. Chellali, T. W. Haynes, S. T. Hedetniemi and T. M. Lewis, On ve-degrees and ev-degrees in graphs, *Discrete Mathematics*, vol. 340, no. 2,(2017), pp. 31-38.

[6] M. Cancan, On ev-degree and ve-degree topological properties of tickysim spiking neural network, *Computational Intelligence and Neuroscience*, vol. 2019, (2019),pp. 1-7.

[7] A. Ahmad, Comparative study of ve-degree and ev-degree topological descriptors for benzene ring embedded in P-type-surface in 2D network, *Polycyclic Aromatic Compounds*, https://doi.org/10.1080/10406638.2020.1834415.

[8] S. Ediz, A new tool for QSPR researches: ev-degree Randic index, *Celal Bayar Universitesi Fen Bilimleri Dergisi*, vol. 13, no. 3,(2017), pp. 615-618.

[9] S. Ediz, On ve-degree molecular topological properties of silicate and oxygen networks, *International Journal of Computer Science Mathematics*, vol. 9, no. 1 , (2018) , pp. 1-12.

[10] Horoldagva, K. C. Das and T. A. Selenge, On ve-degree and ev-degree of graphs, *Discrete Optimization*, vol. 31,(2019), pp. 1-7.

[11] B. Sahin and S. Ediz, On ev-degree and ve-degree topological indices, *Iranian Journal of Mathematical Chemistry*, vol. 9, no. 4, (2018), pp. 263-277.

[12] J. Zhang, M. K. Siddiqui, A. Rauf and M. Ishtiaq, On ve-degree and ev-degree based topological properties of single walled titanium dioxide nanotube, *Journal of Cluster Science*, 2020.

[13] Ediz, S.; Süleyman; Ademir; Şerif, Çiftçi, İdris A. Note on Vertex-Edge Degree Sombor Index of Silicate and Oxygen Networks,(2022), pp. 23-33.

[14] Gowtham, K. J.; New vertex-edge sombor, Nirmala and Misbalance Indices, TWMS J. *App. Eng. Math.* In press.

[15] L. O. Chua and Lin Yang, Cellular Neural Networks: *Theory, IEEE Trans. on CAS*, vol. 35 no. 10, (1988), pp. 1257-1272.

[16] L. O. Chua and Lin Yang, Cellular Neural Networks: Applications, *IEEE Trans. on CAS*, vol. 35 no. 10, (1988), pp. 1273-1290.

[17] A. K. Rizhevsky, I. Sutskever and G. E. Hinton, Image net classification with deep convolutional neural networks, *Advances in Neural Information Processing Systems*, vol. 2,(2012), pp. 1097-1105.

[18] G. Yanan, C. Xiaoqun, L. Bainian and G. Mei, Solving partial differential equations using deep learning and physical constraints, *Applied Sciences*, vol. 10, (2020), pp. 2076-3417.

[19] J. Sirignano and K. Spiliopoulos, A deep learning algorithm for solving partial differential equations, *Journal of Computational Physics*, vol. 375, no. 3, (2018), pp. 1339-1364.

[20] H. Jin, Y. Lian and J. Hua, Learning facial expressions with 3D mesh convolutional neural network, *ACM Transactions on Intelligent Systems and Technology*, vol. 7, (2018), pp. 2157-6904.

[21] N. Kriegeskorte, Deep neural networks: A new framework for modeling biological vision and brain information processing, *Annual Review of Vision Science*, vol. 1, no. 1, (2015), pp. 417-446.

[22] J. B. Liu, Z. Raza and M. Javaid, Zagreb connection numbers for cellular neural networks, *Discrete Dynamics in Nature and Society*, vol. 2020,(2020),pp. 8.

[23] J.B. Liu, Z. Jing, W. Shaohui, M. Javaid and C. Jinde, On the topological properties of the certain neural networks, *Journal of Artificial Intelligence and Soft Computing Research*, vol. 8, no. 4, (2018), pp. 257-268.

[24] M. Imran, M. K. Siddiqui, A. Q. Baig and W. Khalid, Topological properties of cellular neural networks, *Journal of Intelligent & Fuzzy Systems*, vol. 37, no. 3, (2019), pp. 3605-3614.

[25] V. Bhambhani and H. G. Tanner, "Topology optimization in cellular neural networks," in 49th IEEE Conf. on Decision and Control, Atlanta, GA, USA, IEEE, 2010.

[26] F. Ejaz, M. Hussain, H. Almohamedh, K. M. Alhamed, R. Alabdan et al., "Dominating topological analysis and comparison of the cellular neural network," *Mathematical Problems in Engineering*, vol. 2021, Article ID 6613433, (2021), 9 pages. https://doi.org/10.1155/2021/6613433

[27] Sadia Husain, Muhammad Imran, Ali Ahmad, Yasir Ahmad and Kashif Elahi, A Study of Cellular Neural Networks with Vertex-Edge Topological Descriptors, *Computers, Materials & Continua*, vol. 70, no. 2, (2022), pp. 3433-3447.